Review of Methods to Produce Round Beams

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Content

I. Round Beams – Why?

„ A significant fraction of the beamline users at Swiss light source (SLS) prefer “round beam“ rather than flat beam, ...“, M. Aiba, et al., TUPJE045, IPAC2015, Richmond, VA, USA

What users prefer most is a beam limited by fundamental effects in all dimensions – diffraction or Fourier limit.

IPAC17

Spectral brilliance and coherent fraction

Liu Lin:“Towards Diffraction Limited Storage Ring Based Light Sources“

LNLS sirius

- Spectral brilliance: Flux density in phase space

$$B(\lambda) \propto \frac{F(\lambda)}{\left(\epsilon_{x,e^-} \otimes \epsilon_r(\lambda)\right)\ \left(\epsilon_{y,e^-} \otimes \epsilon_r(\lambda)\right)}$$

Photon flux [photons/s/0.1% bw]

electron beam emittance

photon limiting emittance

$$\epsilon_r = \sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$$ for Gaussian beam

$$\epsilon_r = \sigma_r \sigma_{r'} \approx \frac{\lambda}{2\pi}$$ for undulator beam

- Coherent fraction for undulator radiation

$$f_{coh} = \frac{(\lambda/2\pi)^2}{\left(\epsilon_{x,e^-} \otimes \epsilon_r(\lambda)\right)\ \left(\epsilon_{y,e^-} \otimes \epsilon_r(\lambda)\right)}$$

- Diffraction limited storage ring

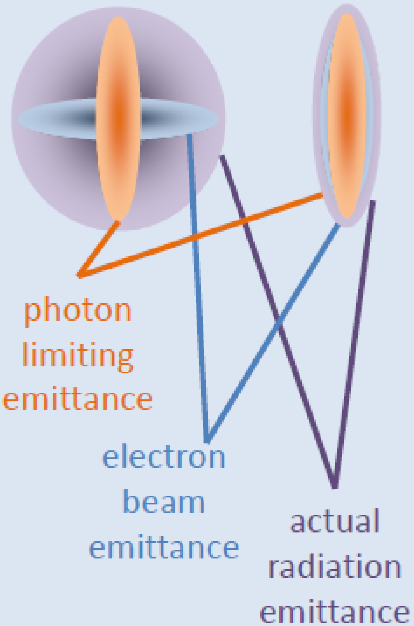
$$\epsilon_{x,y} \approx \epsilon_r(\lambda) = \frac{\lambda}{2\pi}$$

$\varepsilon_{x,y} \approx$ 100 pm.rad
diffraction limit for 2 keV

$\varepsilon_{x,y} \approx$ 20 pm.rad
diffraction limit for 10 keV

Diffraction Limit: low emittance is not all! Phase space matching

Electron beam and radiation phase-space



Matching condition

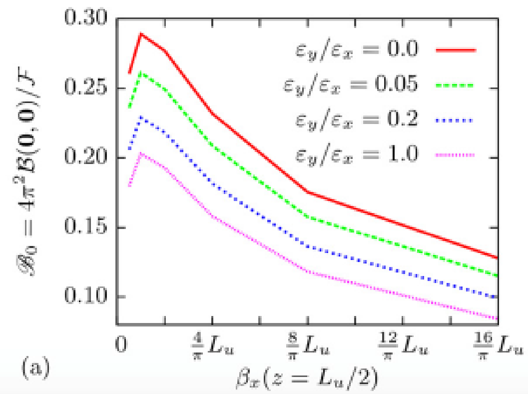
$$\beta_e = \frac{\sigma_e}{\sigma'_e} = \frac{\sigma_r}{\sigma'_r}$$

Highest brilliance from undulator of length L is achieved when

$$\beta_{x,y}^{opt} \approx \frac{L}{\pi} \qquad \beta_{x,y}^{opt} \sim 1 - 2m$$

Ryan R. Lindberg and Kwang-Je Kim (2015)

COMPACT REPRESENTATIONS OF PARTIALLY … Phys. Rev. ST Accel. Beams **18,** 090702 (2015)



IV. CONCLUSIONS

In this paper we have described three different coherent mode representations of partially coherent undulator radiation. We began with the well-known Gaussian-Schell decomposition in terms of Gauss-Hermite modes, which is valid provided the electron beam emittance is much larger than the natural radiation emittance $\lambda/4\pi$. In this largely incoherent case the specifics of the single-electron undulator field are unimportant. We then refined our analysis to include the situation when the electron beam emittance ε_y in one direction is arbitrary, and found that the modes along y are determined by solving a matrix

Figure 8(a) shows that the coherence is maximized when $\beta_x \approx L_u/\pi$ (or $\hat{\beta}_x \approx 1$), which indicates that the "natural" Rayleigh range of undulator radiation $Z_R \approx \beta_x \approx L_u/\pi$. Unfortunately, it is nearly impossible for lattice designers to make the beta functions in both x and y be simultaneously that small, and typically $\beta_x > 3L_u/\pi$.

very similar: the profiles when β_x is increased by a factor of 16 can be approximately obtained by multiplying those in Fig. 8(b) by 1.3. Figure 8(c) plots the profiles along x as we vary β_x. Each plot is approximately Gaussian, and we see that the angular spread decreases as β_x increases. More careful inspection shows that the width of the angular

Liu Lin:"Towards Diffraction Limited Storage Ring Based Light Sources"

I. Round Beams – Why?

„ A significant fraction of the beamline users at Swiss light source (SLS) prefer “round beam“ rather than flat beam, ...“, M. Aiba, et al., TUPJE045, IPAC2015, Richmond, VA, USA

What users really prefer most is radiation optimized for their own experiment.

The low emittance will be spoiled from emittance growth due to Intra Beam Scattering (IBS) in case of too high particle density in the bunch – mitigated by longer bunches and round beams.

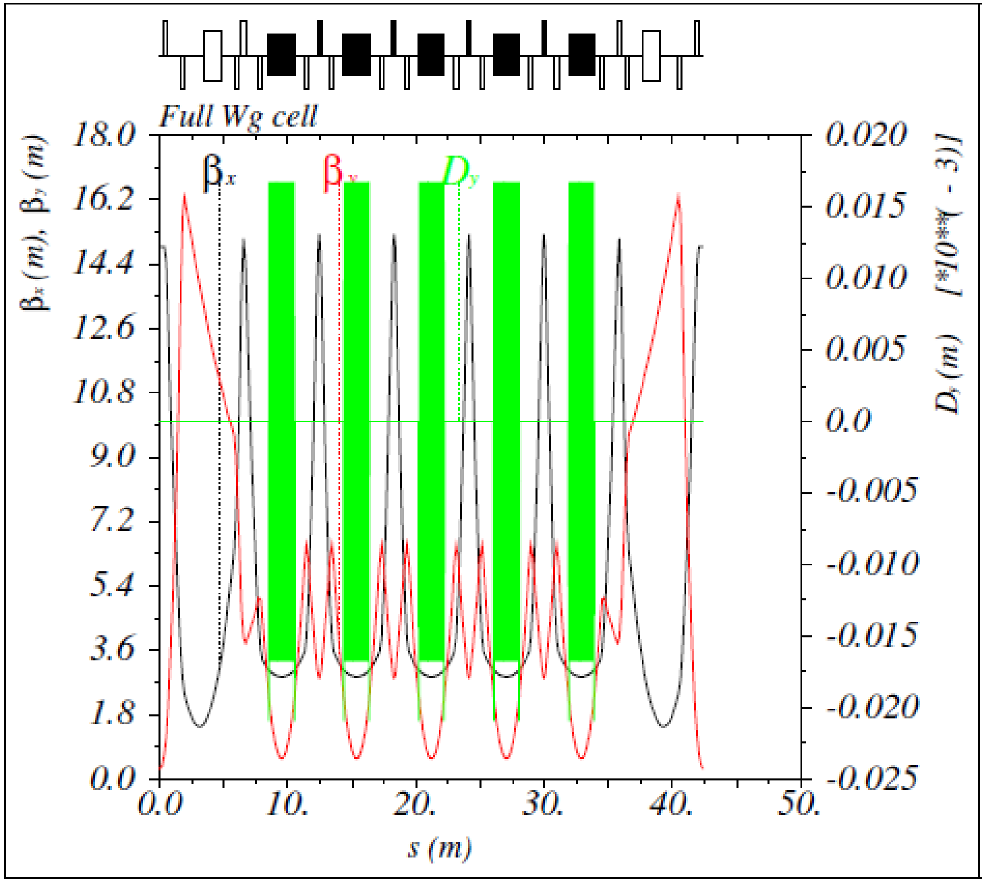
Ultimate synchrotron radiation source with horizontal field wigglers

A. Bogomyagkov, E. Levichev, P. Piminov, S. Sinyatkin

Budker Institute of Nuclear Physics
Novosibirsk

Low Emittance Rings 2014 Workshop
17-19 September 2014 INFN-LNF

Straight section: damping wigglers with horizontal field



Wigglers with horizontal field:

$B = 2.3$ T

$\lambda = 4.8$ cm

$N_\lambda = 42$

$L_{wiggler} = 2.04$ m

$N_{total} = 20$

$L_{total} = 40.8$ m

Parameters of the ring

$$Ring = 4 \times 6 \times \left[5 \times FiveCell + Straight\right]$$

20 straight are sections empty

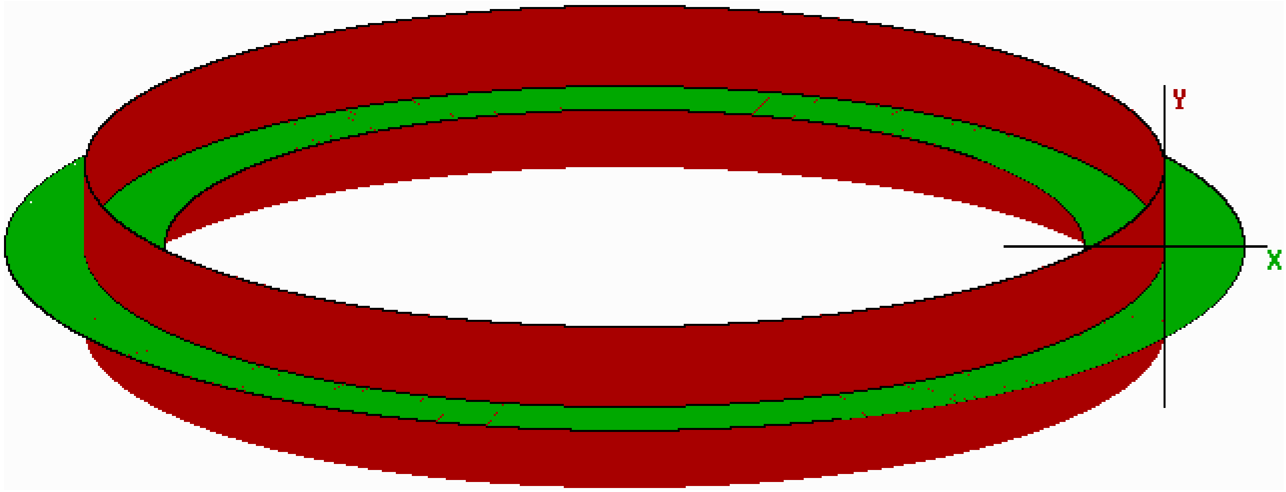
4 straight sections are occupied by damping wigglers

	Wigg OFF	Wigg ON
Energy, GeV	3	
Circumference, m	1379	
Chromaticity h/v	-184/-251	
Betatron tunes h/v	84.52/91.772	
Horizontal Emittance, pm rad	64	3
Vertical Emittance, pm rad	0.6	8.6
Energy spread	4×10^{-4}	1.2×10^{-3}
Momentum compaction	7.8×10^{-5}	7.8×10^{-5}
Damping times h/v/s, msec	210/210/105	10/10/5
Wiggler field, T	0	2.33

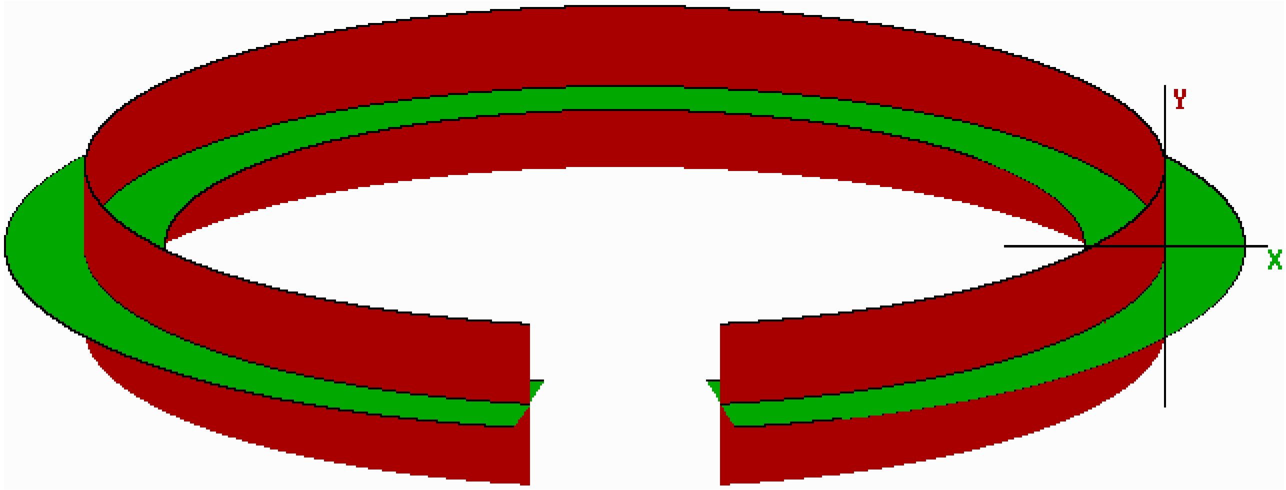
III. Round Beam in Möbius Accelerator

In the Möbius Accelerator transverse particle coordinates are exchanged every turn by a set of skew quadrupole magnets sharing the natural emittance equally among the two planes. (R. Talman, PRL 74, 1590 (1995) and M. Aiba, et al., TUPJE045, IPAC2015, Richmond, VA, USA)

Uncoupled storage ring:



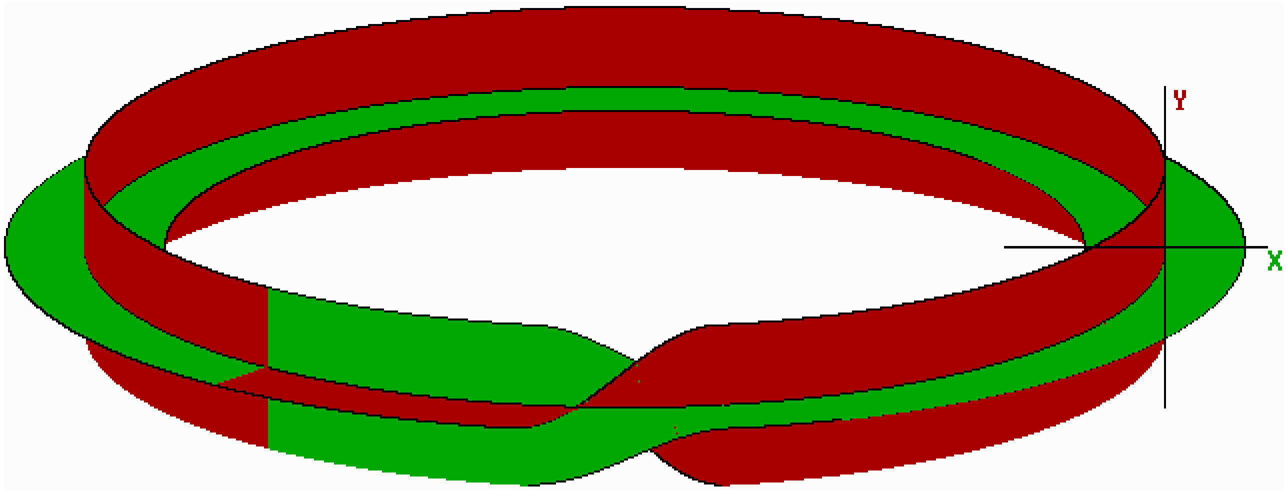
Uncoupled storage ring:



insertion

III. Round Beam in Möbius Accelerator

Orbit and optics repeat every second turn:



rotator – complete exchange of horizontal and vertical motion

transverse particle coordinates are exchanged every turn by a set of skew quadrupole

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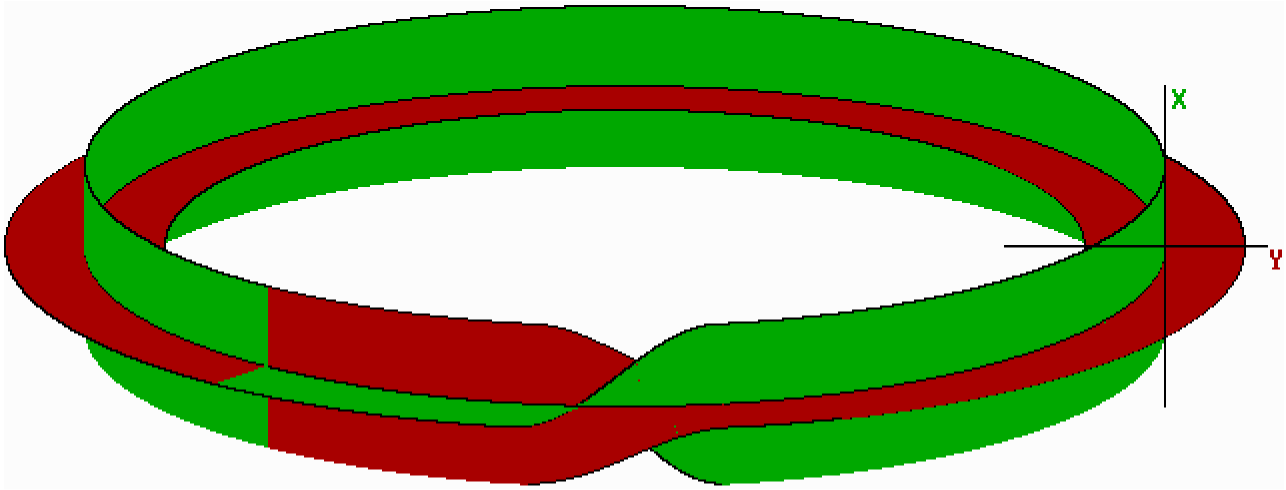


rotator – complete exchange of horizontal and vertical motion

transverse particle coordinates are exchanged every turn by a set of skew quadrupole

Award-winning Norwegian architectural firm, Snøhetta, unveiled an innovative proposal for the Max-Lab in Lund, Sweden.
The circular shape is twisted and raised to create a dynamic form based on a Möbius strip that becomes an actual volume, not just a ribbon.

In the Möbius Accelerator transverse particle coordinates are exchanged every turn by a set of skew quadrupole magnets sharing the natural emittance equally among the two planes.

Off-axis injection impossible with this really strong coupling of the horizontal and vertical plane.

Comparison of these two techniques:

Technical Approach	Injection	Emittance Control	Complexity
Radial Damping Wiggler	Off-axis	yes	large
Möbius Accelerator	On-axis	no	challenging

E. Wilson „Linear Coupling“, CERN 85-19, p. 114 with time dependent skew quadrupole:

$$\ddot{x}+\omega_x^2 x=-y\cdot k\cdot(e^{i\omega t}+e^{-i\omega t})/2$$
$$\ddot{y}+\omega_y^2 y=-x\cdot k\cdot(e^{i\omega t}+e^{-i\omega t})/2$$

Ansatz – small coupling:

$$x(t)=X(t)\cdot e^{i\omega_x t}$$
$$y(t)=Y(t)\cdot e^{i\omega_y t}$$

X(t) and Y(t) are slowly varying functions – second time derivatives as well as fast

$$2i\omega_x\dot{X}=-Y\cdot k\cdot\left[e^{i(\omega-\Delta\omega)t}+\cancel{e^{-i(\omega+\Delta\omega)t}}\right]$$
$$2i\omega_y\dot{Y}=-X\cdot k\cdot\left[e^{-i(\omega-\Delta\omega)t}+\cancel{e^{i(\omega+\Delta\omega)t}}\right]$$

oscillating terms are ignored

$$\Delta\omega=\omega_x-\omega_y$$

coupled first order turned into uncoupled second order differential equation:

$$\ddot{X}-i(\Delta\omega-\omega)\dot{X}+\frac{k^2}{16\omega_x\omega_y}X=0$$

on resonance – the fast oscillation *x(t)* shows a harmonic modulation and beating with energy exchange to the vertical plane occurs

General resonance condition:

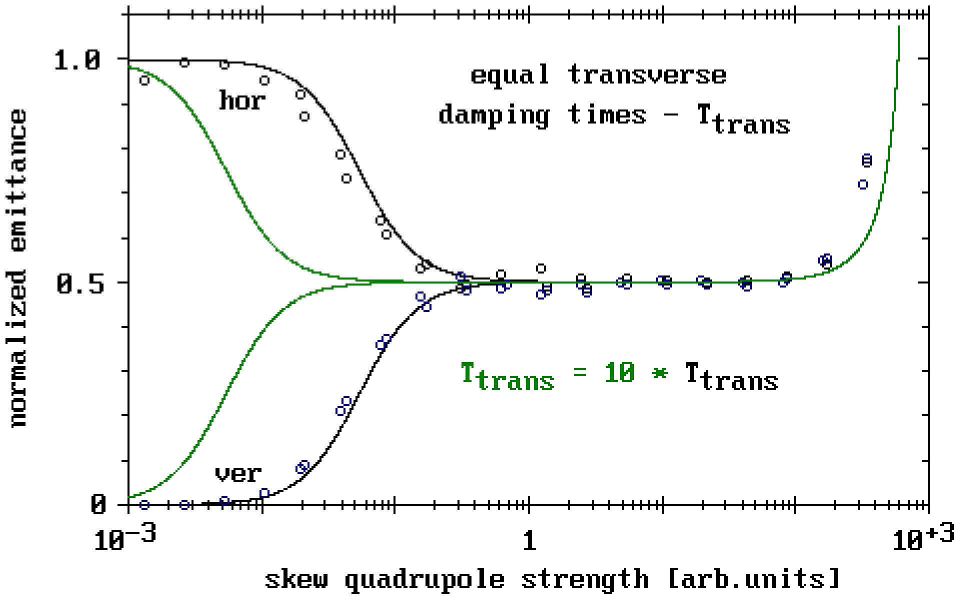
$$Q_x-Q_y=n\pm\omega/\omega_0$$

with the revolution frequency, ω_0, and the frequency of the skew gradient, ω.
Identical results for coupling created by constant or time dependent fields in solenoids.

linear coupling due to skew quadrupole gradient:

Q_x-Q_y=n, n=integer

on resonance emittance sharing - $\varepsilon_y=\varepsilon_x=\varepsilon_0/2$
with equal damping times, $T_x=T_y$, in both planes



Comparison of solutions from multi particle tracking and first modeling attempts with analytical solutions based on moment mapping.

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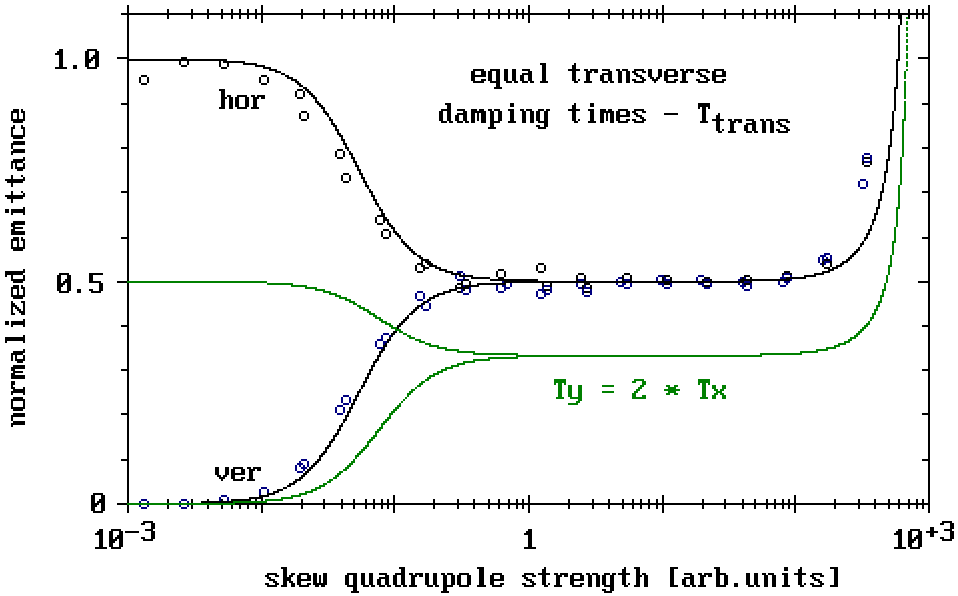

Comparison of solutions from multi particle tracking and first modeling attempts with analytical solutions based on moment mapping.

With $T_x=T_y/2$ and on resonance **$\varepsilon_y = \varepsilon_x = 2/3\ \varepsilon_0$**

linear coupling due to skew quadrupole gradient:

Q_x-Q_y=n, n=integer

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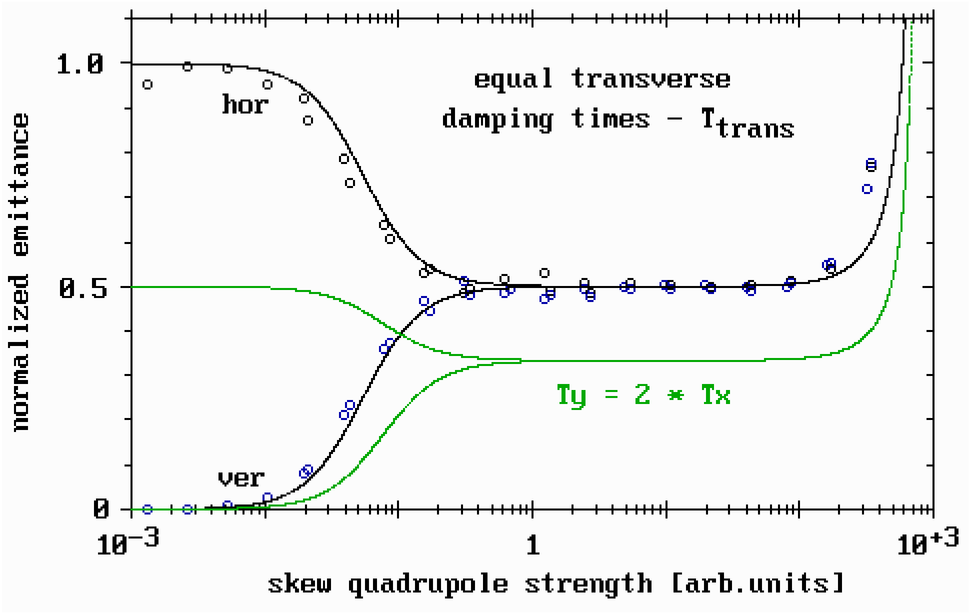

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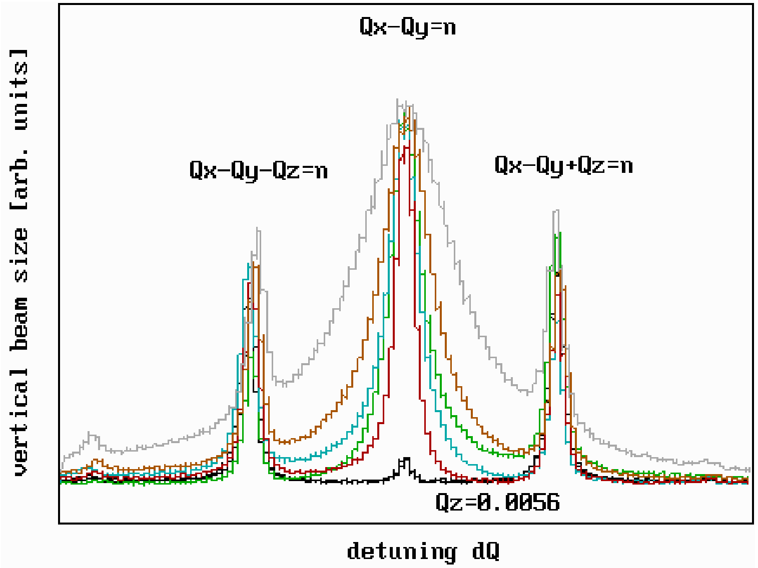


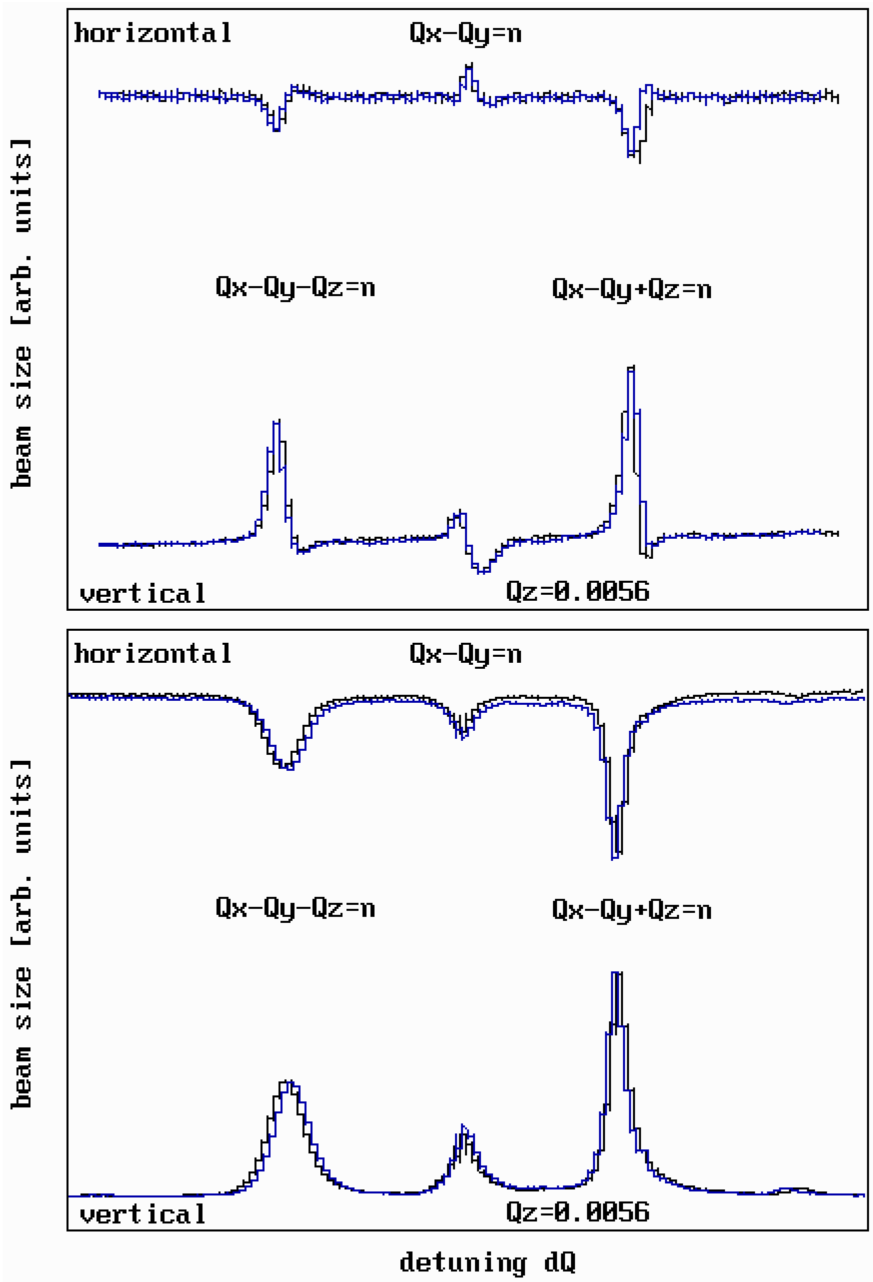
Compensation of the coupling resonance in the BESSY II storage ring – as expected: damping dominates for very small coupling coefficients, and width depends on coupling strength: „power broadening“, will be helpful later on

Compensation of linear coupling:

Low beam current – 7 mA in 320 bunches

Max. current 250 mA – hybrid fill pattern: train of 320 bunches and 5 bunches with 4 mA each

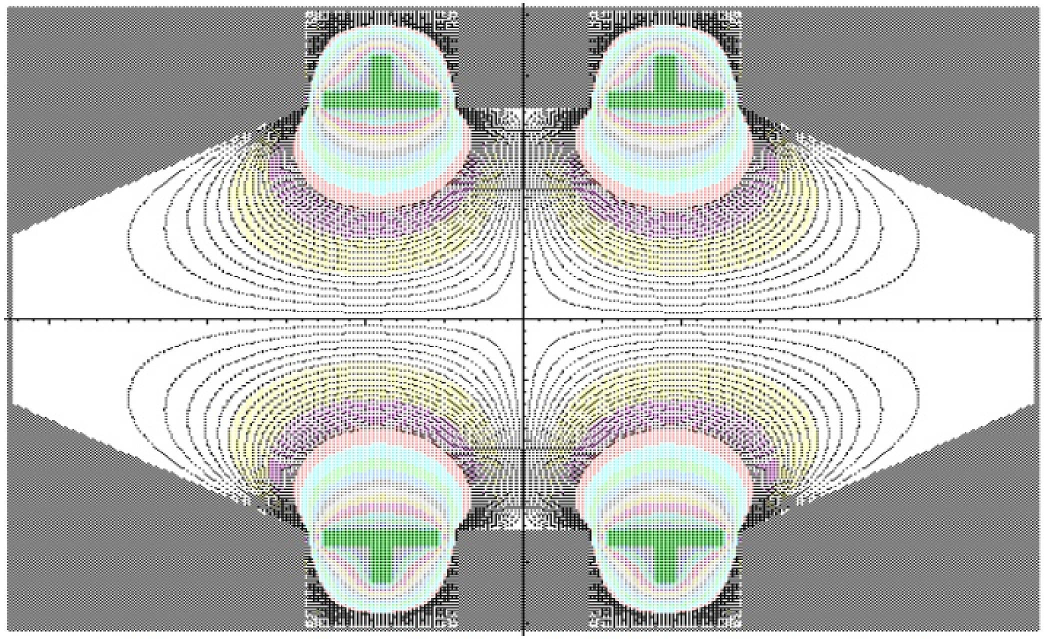
Compensation not spoiled by intensity – some collective effects visible



For better control of the coupling and in case the storage ring can not be operated at the coupling resonance the resonance can be excited artificially. With a time dependent sinusoidal varying skew gradient the resonance condition is:

$$Q_x - Q_y = n \pm \omega/\omega_0$$

with the revolution frequency, ω_0, and the frequency of the skew gradient, ω.



Skew quadrupole-like field distribution in the centre of the stripline arrangement.

Neighboring currents in opposite directions.

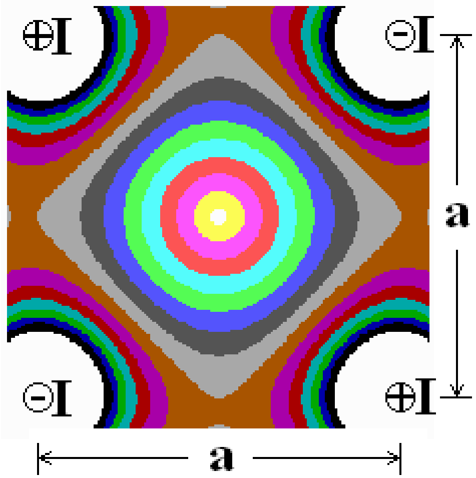
Full coupling and emittance sharing achievable –

- little power broadening,
- sensitive to tune jitter.

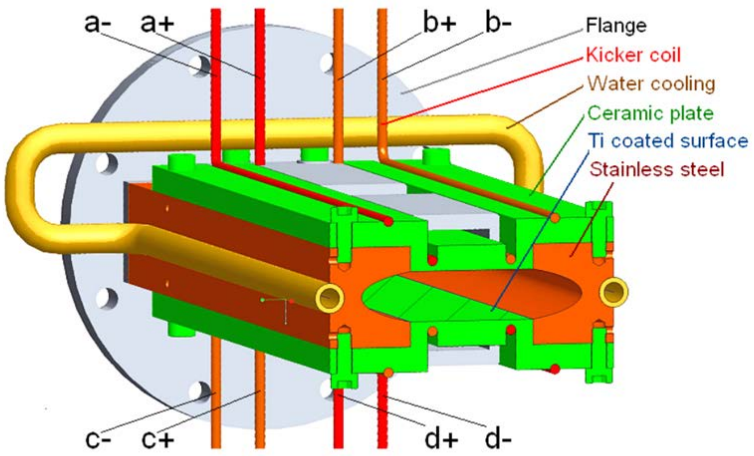
P. Kuske, R. Görgen, EPAC 2002, WEPLE015

The required frequency, F_{sq}, for the skew quadrupole is on the order of 100 kHz. Striplines are not really required. Simpler design could look like this:

skew quadrupole with four wire arrangement and currents flowing in alternating directions



$$\left|\frac{\partial B_x}{\partial x}\right| = \frac{4 \cdot \mu \cdot I}{\pi \cdot a^2} = \frac{1.6 \cdot 10^{-6} \cdot I[A]}{a^2[m^2]} \; [T/m]$$

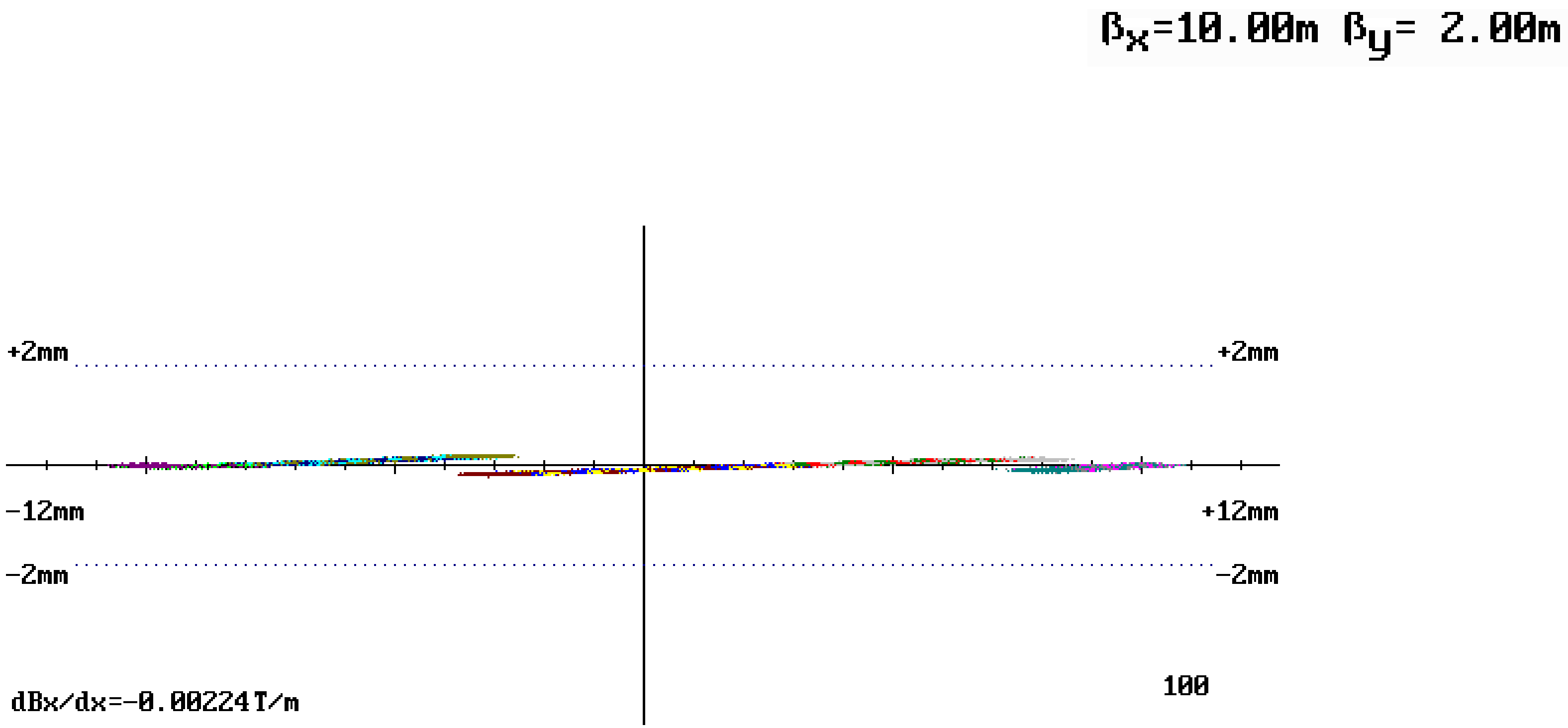


quite similar to our non-linear injection kicker magnet

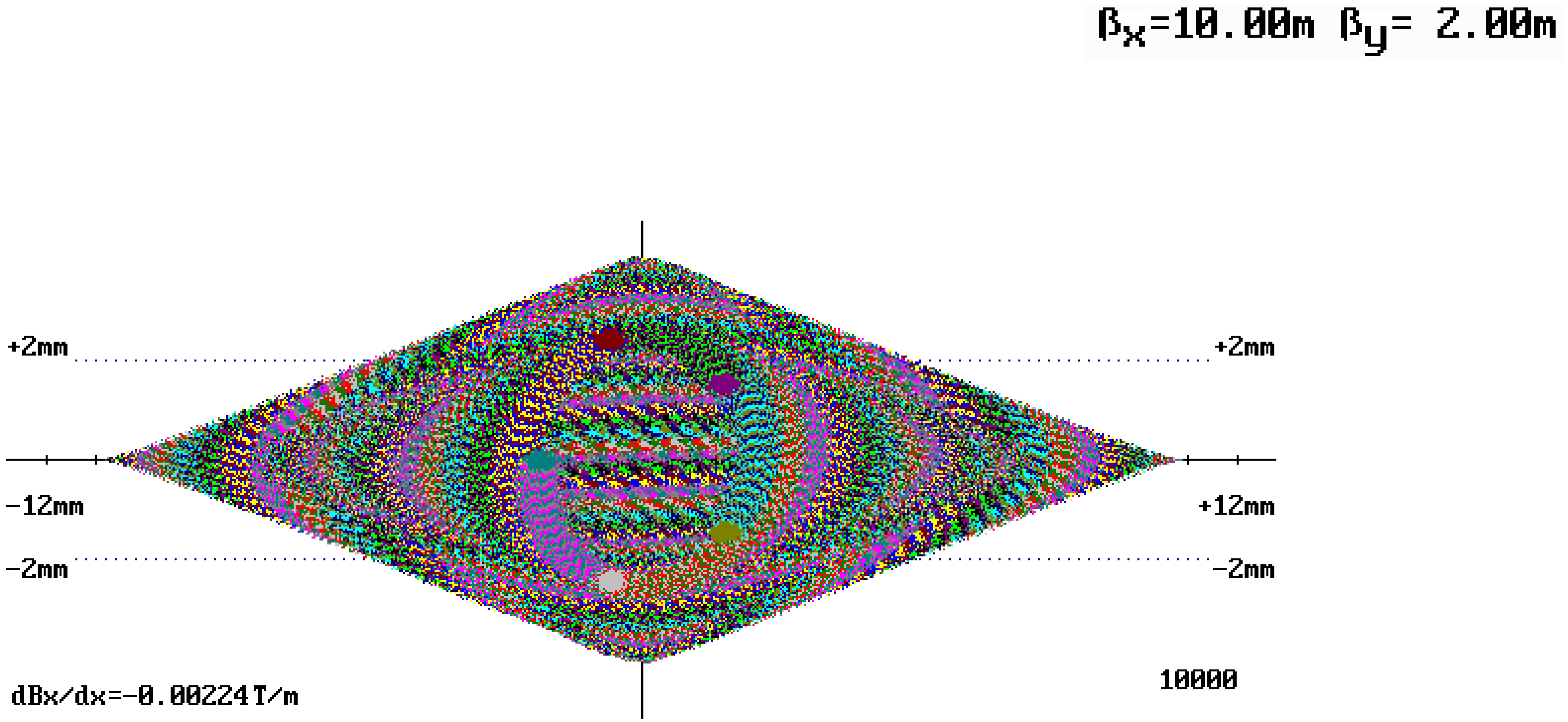
A time dependent solenoid is maybe even simpler to construct.

VI. EMITTANCE SHARING – Static Skew Quadrupole Magnet

Beam injected at +10 mm – the first 100 turns

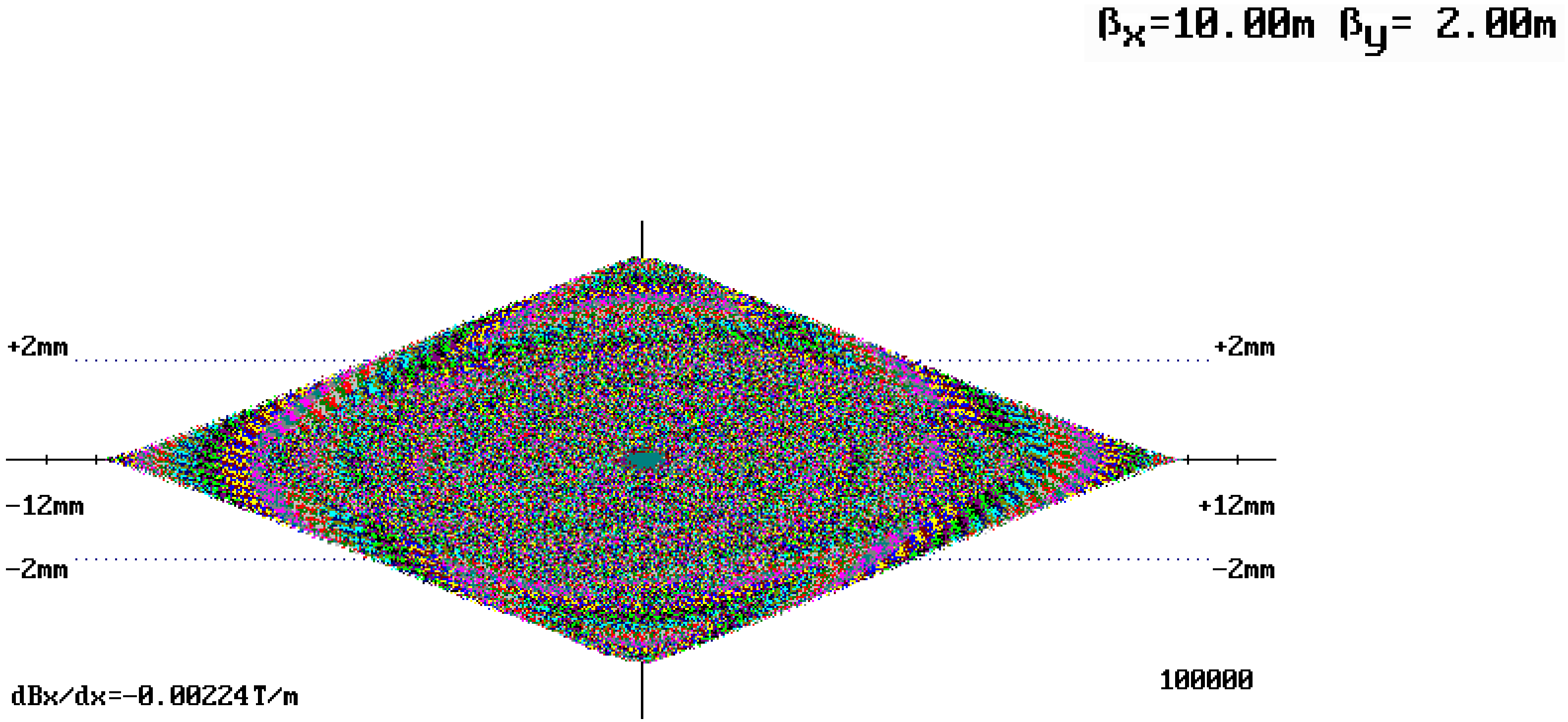


Beam injected at +10 mm – the first 10000 turns



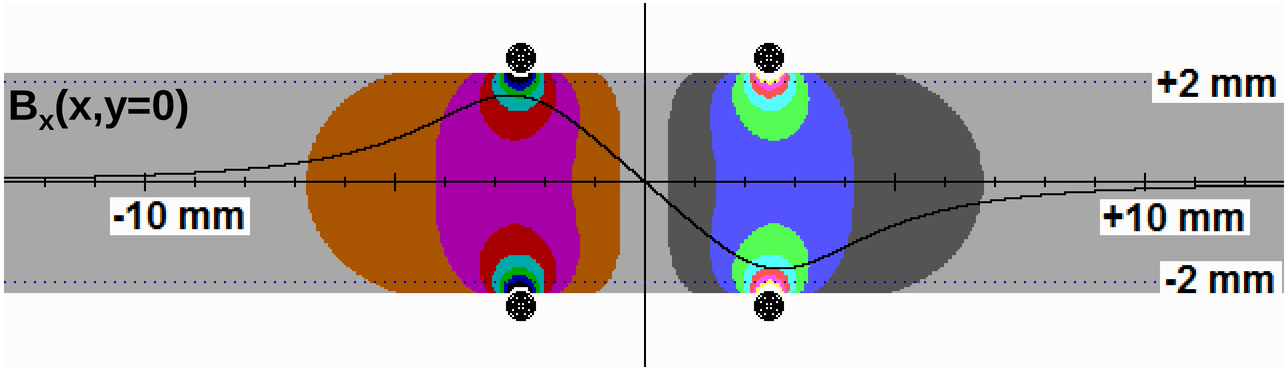
VI. EMITTANCE SHARING – Static Skew Quadrupole Magnet

Beam injected at +10 mm – the first 100000 turns

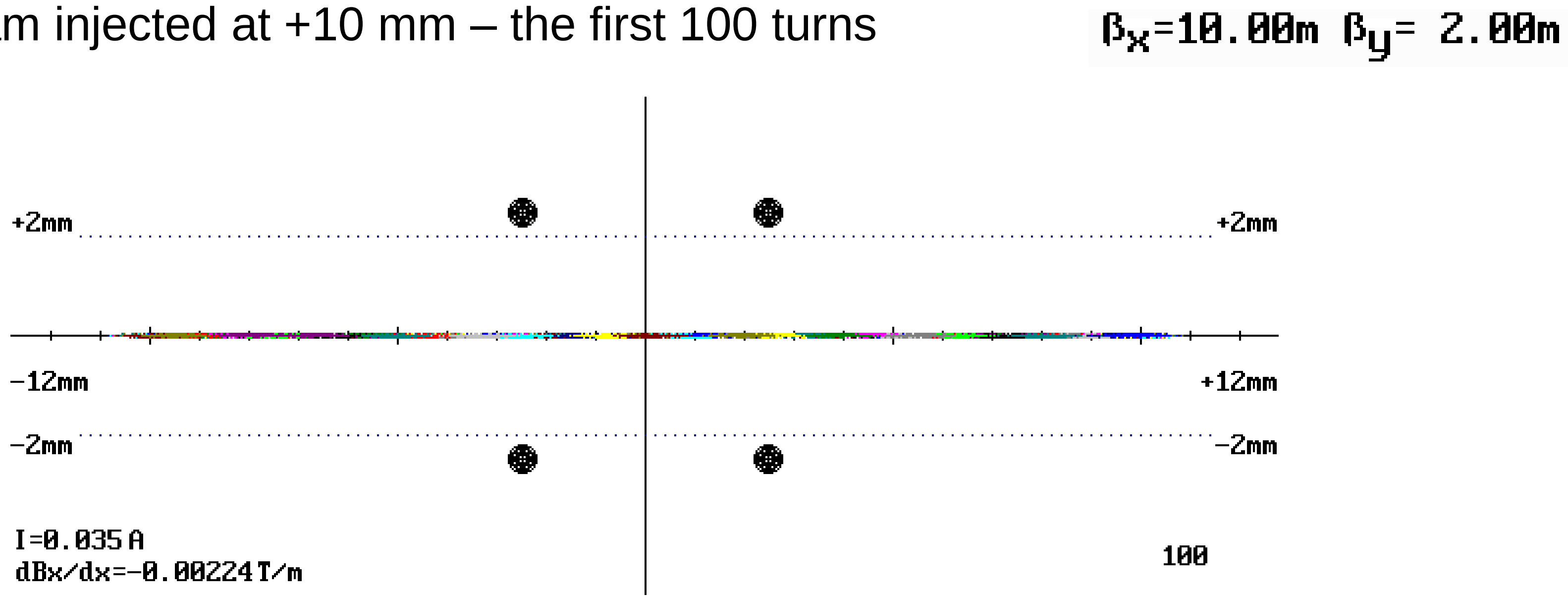


Required acceptance exceeds vertical acceptance – with perfect skew quadrupole magnet

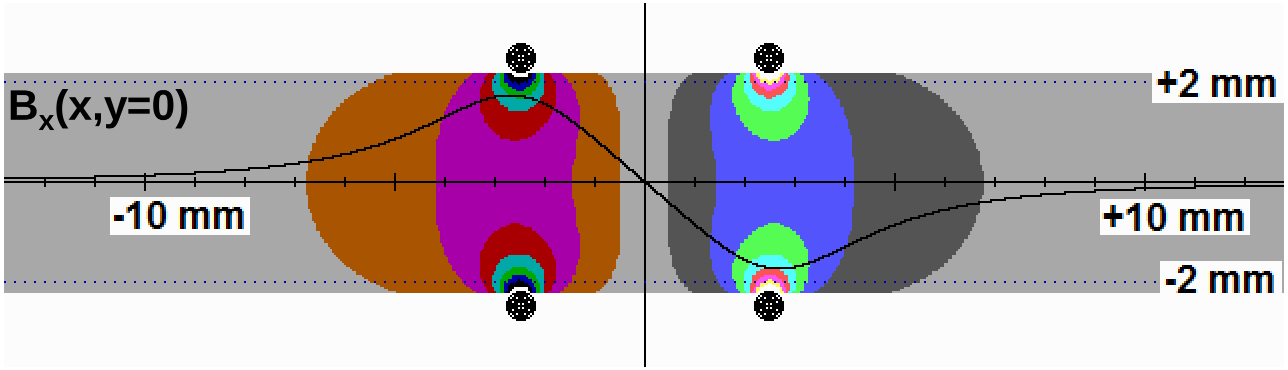
Required acceptance – non-linear skew quadrupole magnet:



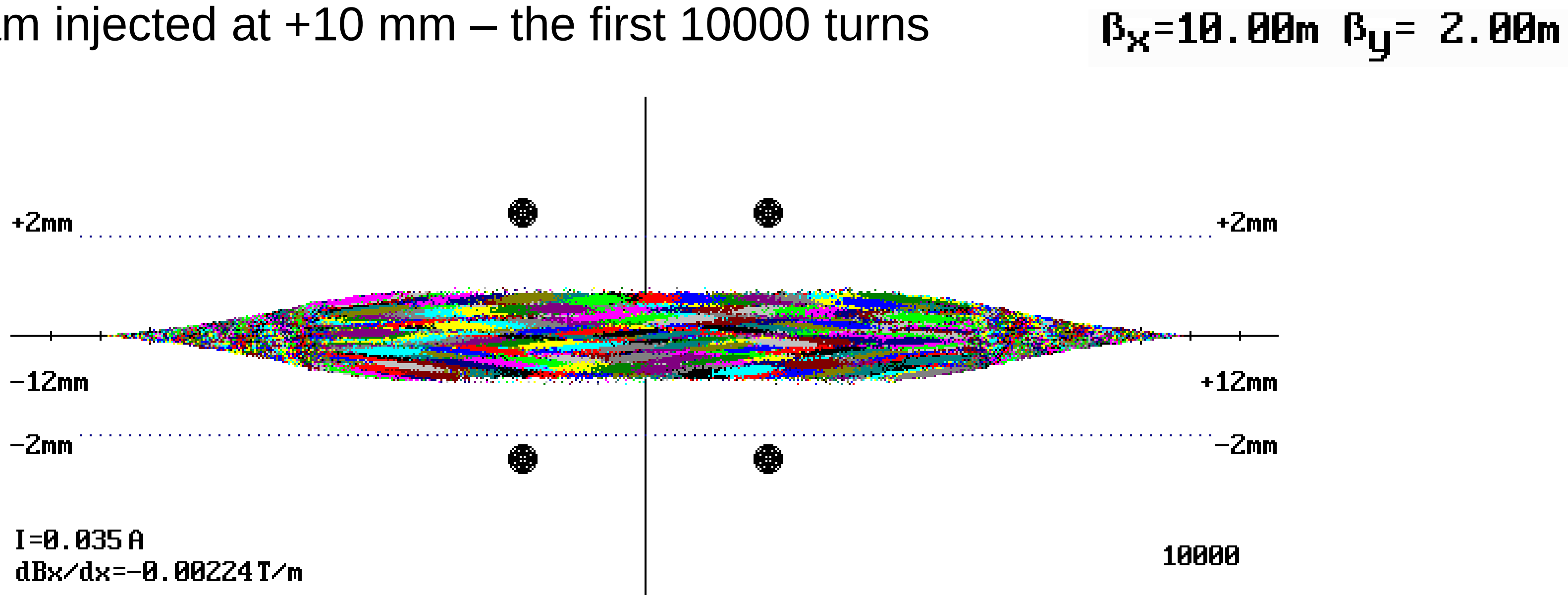
Beam injected at +10 mm – the first 100 turns



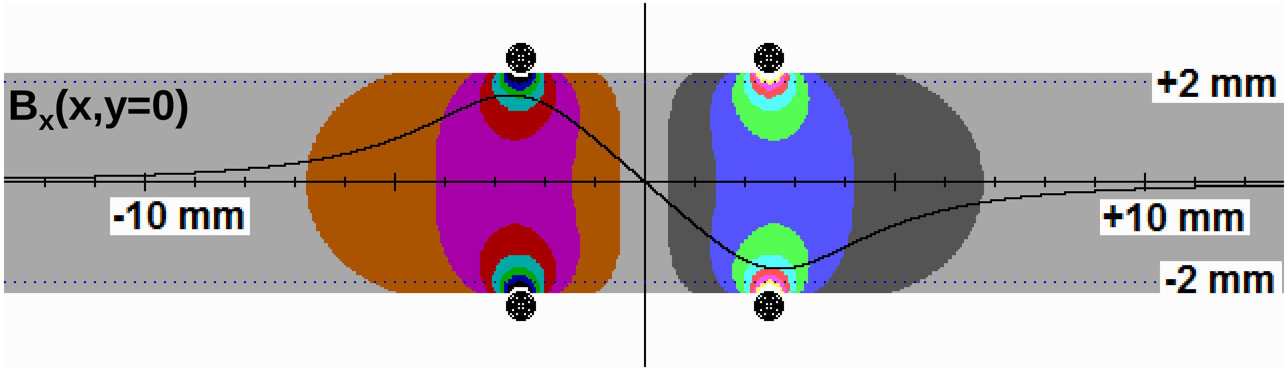
Required acceptance – non-linear skew quadrupole magnet:



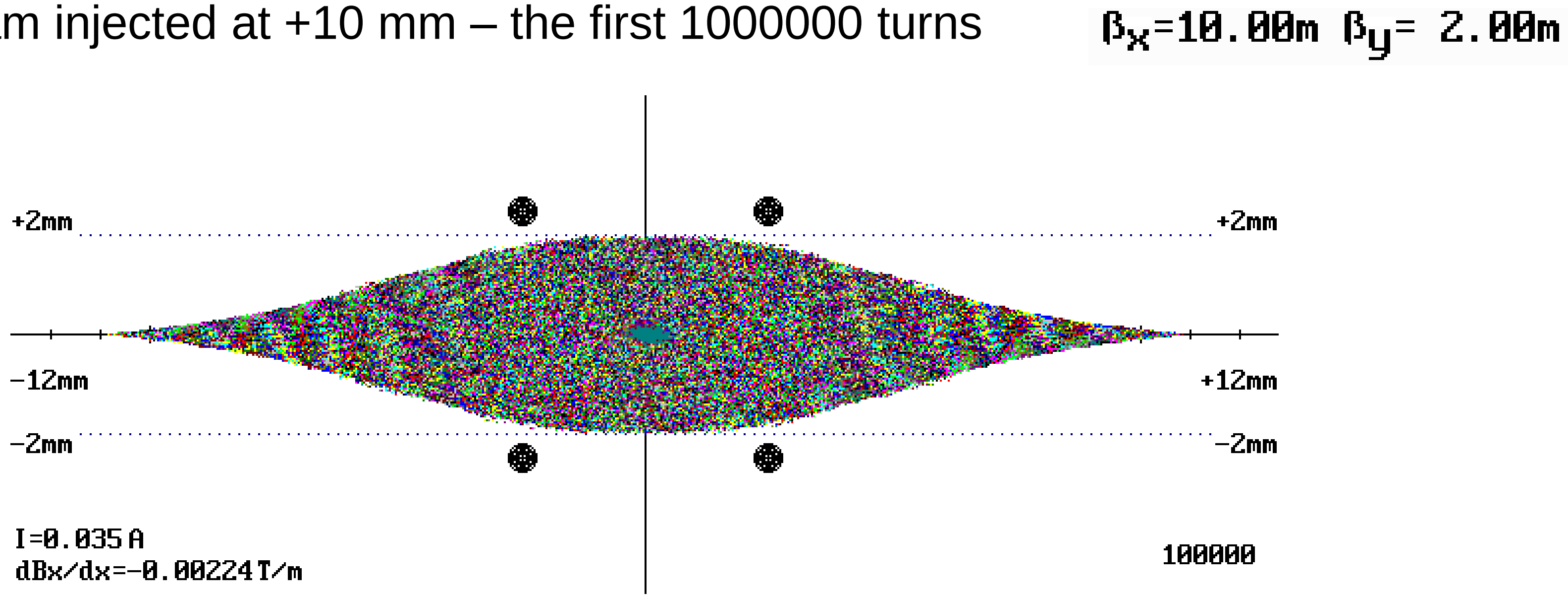
Beam injected at +10 mm – the first 10000 turns



Required acceptance – non-linear skew quadrupole magnet:

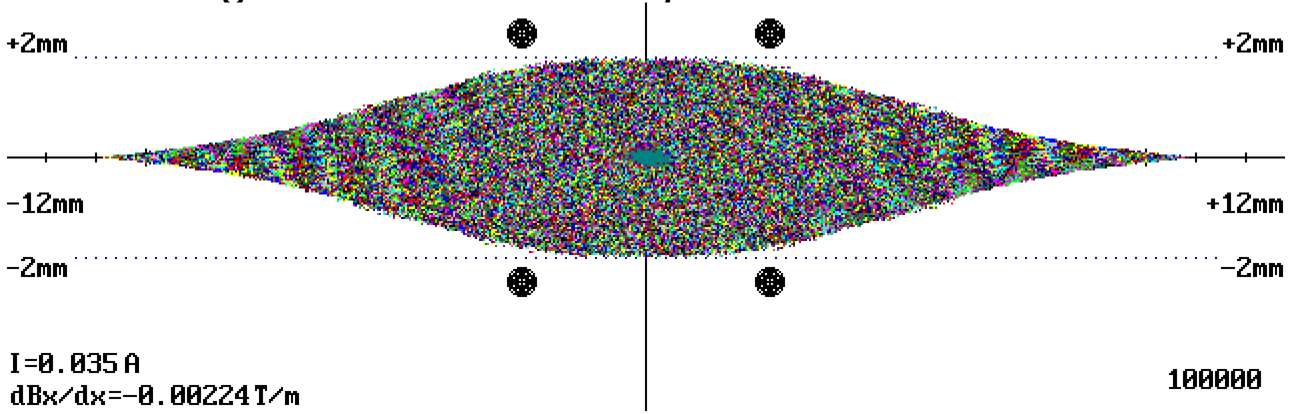


Beam injected at +10 mm – the first 1000000 turns



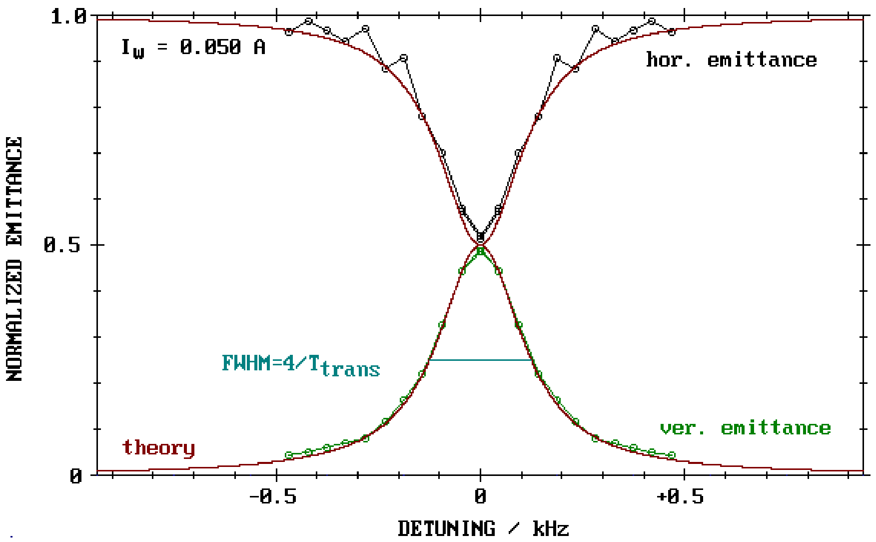
Required acceptance fits vertical acceptance – with non-linear skew quadrupole magnet

Level of excitation as large as not to cause injection losses

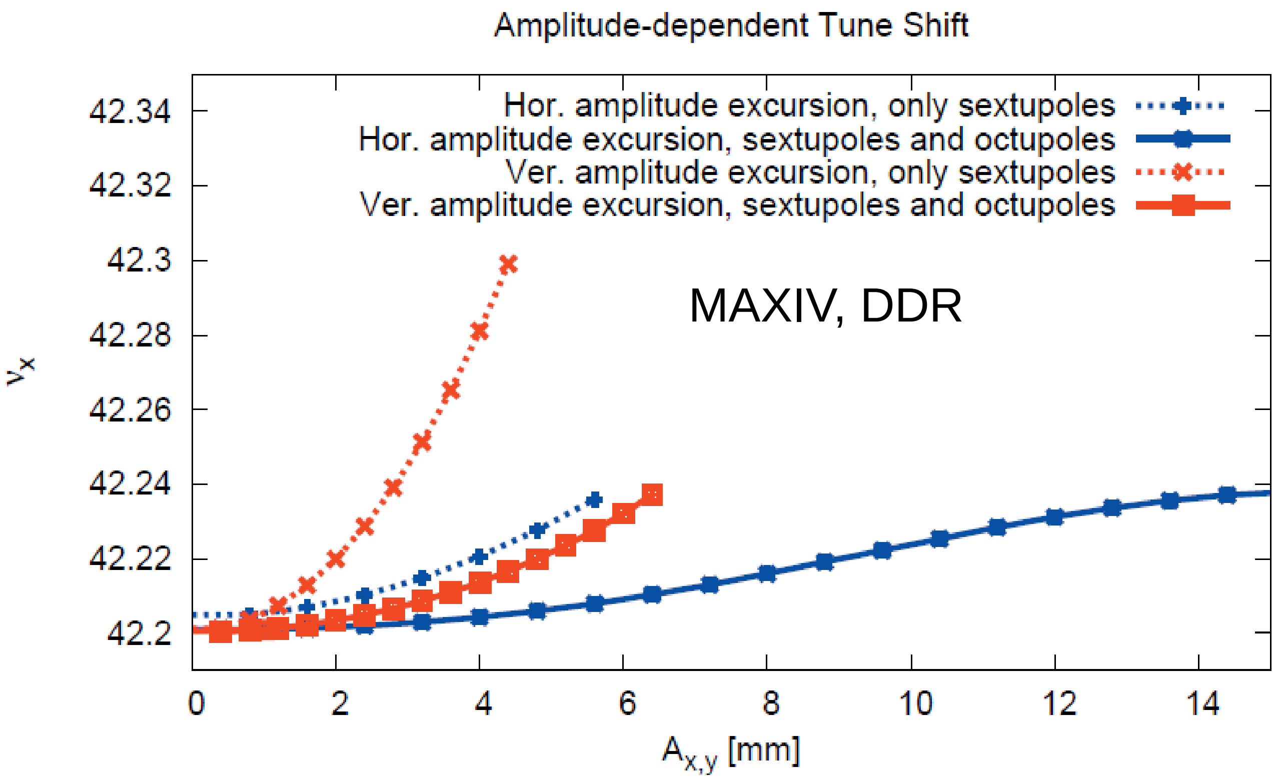


Level of excitation will still be too small to broaden the coupling resonance considerably

Small resonance width – stability of the tunes sufficient? → active tune stabilization

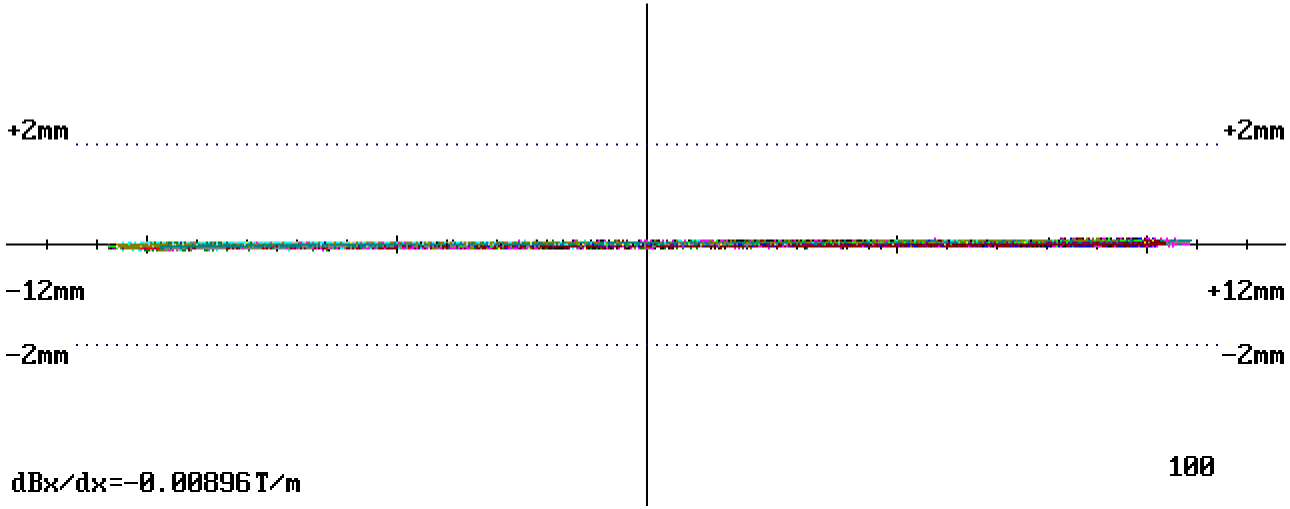


Tune shift with amplitude would help – resonance condition only fulfilled for small amplitudes, stronger excitation could be used



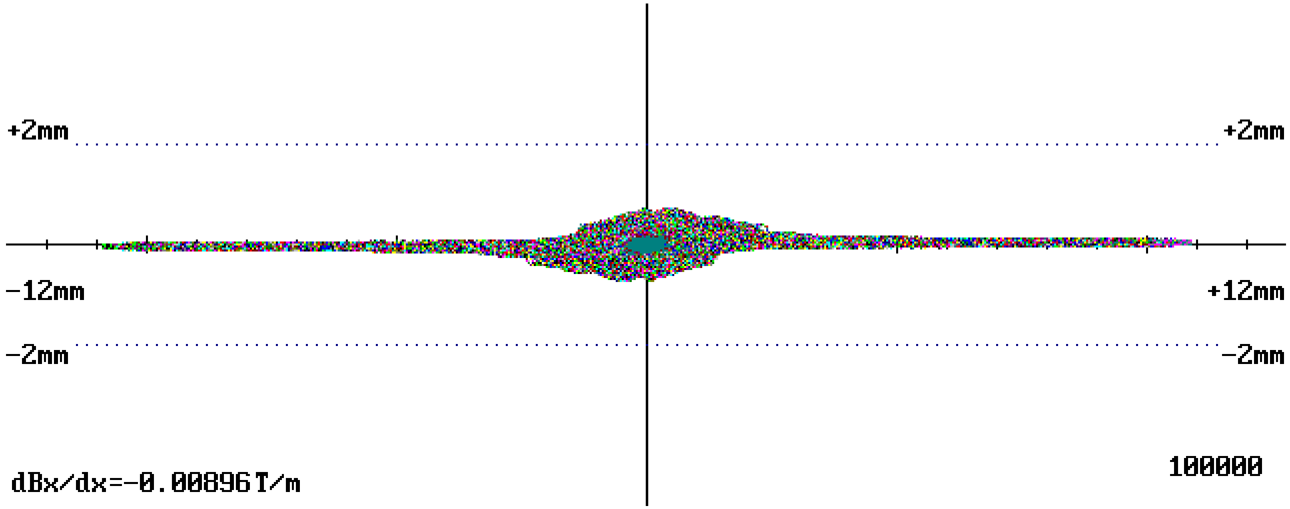
$\Delta\nu_x$=0.02 → ΔF_x=11.4 kHz – much larger than the natural resonance width $\cong 4/\tau_{trans}$ or width due to non-linear chromatic effects

Beam injected at +10 mm – with tune shift with amplitude - the first 100 turns



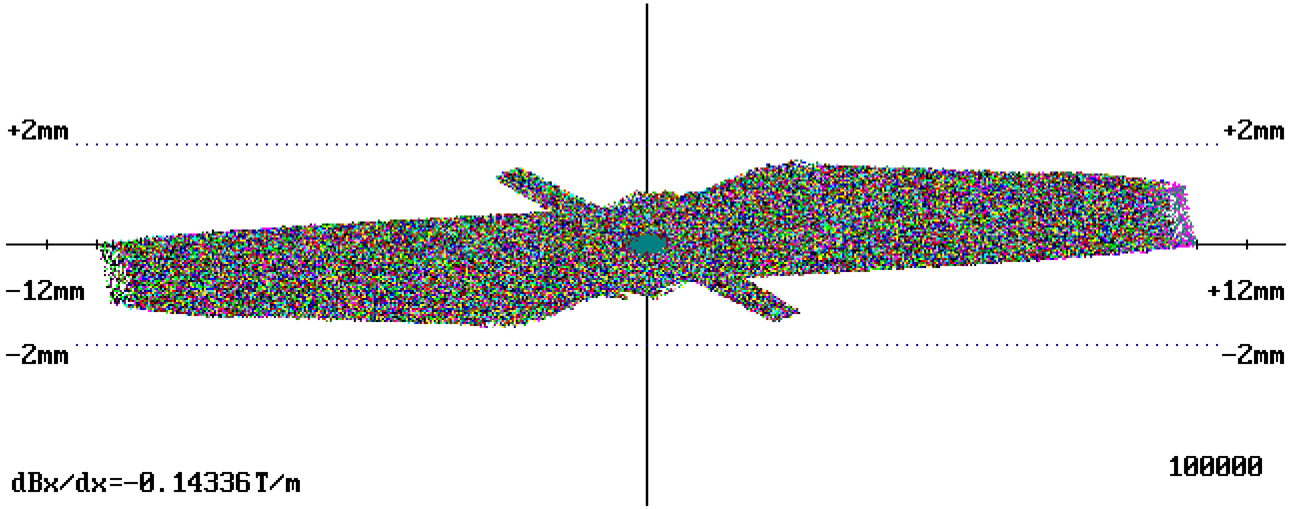
4 times larger skew gradient – fast filamentation of the injected beam due to non-linearity which creates $2{\cdot}10^{-2}$ tune shift for 10mm horizontal amplitude

Beam injected at +10 mm – with tune shift with amplitude - over100000 turns



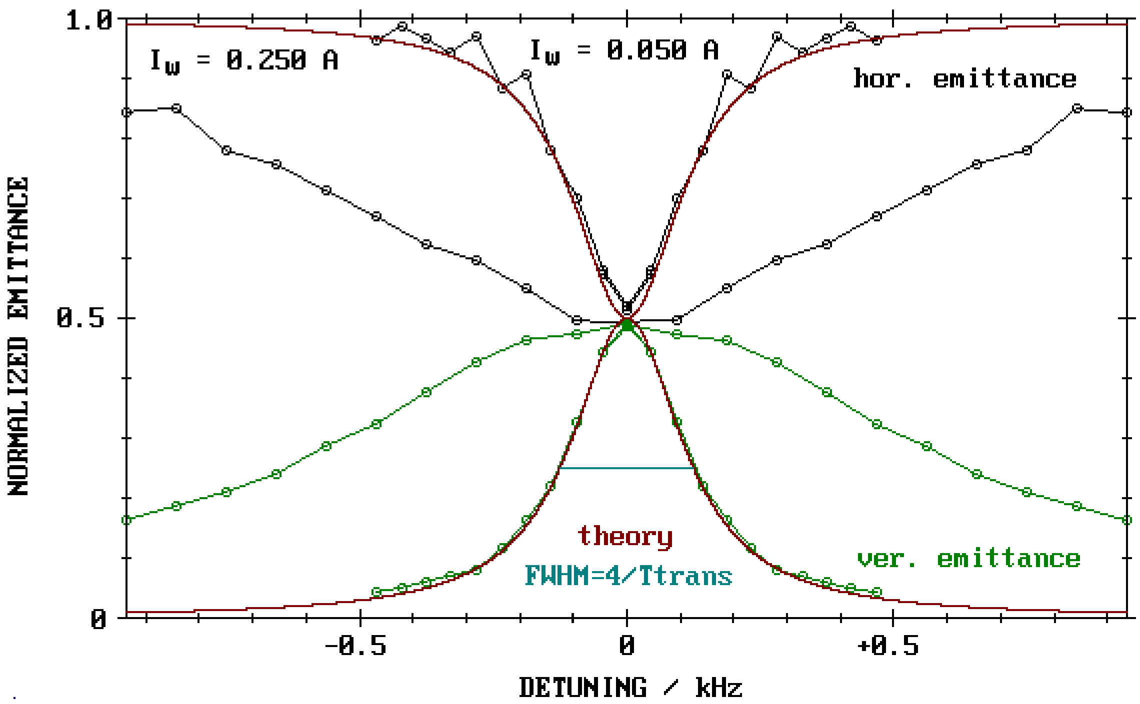
4 times larger skew gradient – relaxed aperture requirement due to tune shift with amplitude

Beam injected at +10 mm – with tune shift with amplitude – over 100000 turns



Even with a much larger skew gradient the injected beam remains within the vertical acceptance

The resonant coupling sets in for small horizontal oscillation amplitude. Stronger skew gradients will not cause losses of injected particles and the “power broadening” can be made as large as desirable and acceptable by the amplitude dependent tune shift. Tune drift and jitter become less important.



This could be tried out at MAXIV with the tune of the ring set to the coupling resonance and the result would be equal emittances in both planes of ~200 pm·rad

Four techniques for the production of round beams have been proposed:

- **Radial wiggler fields** (A. Bogomyagkov, et al., „Ultimate synchrotron radiation source with horizontal field wigglers“, LER Workshop, September 2014, Frascati, Italy)
- **The Möbius accelerator** (R. Talman, PRL 74, 1590 (1995) and M. Aiba, et al., TUPJE045, IPAC2015, Richmond, VA, USA)
- **Artificial excitation of the coupling resonance with special magnet**
- **Sitting on the coupling resonance and tune shift with amplitude**

These techniques require careful tune stabilization or adjustment of excitation frequency and strength of coupling fields (skew gradients or solenoid field)

Technical Approach	Injection	Emittance Control	Complexity
Radial Damping Wigglers	off-axis	Yes	large
Möbius Accelerator	on-axis	no	challenging
Coupling Resonance Excitation	off-axis	(no)	moderate
On Coupling Resonance	on-axis off-axis, tune shift with amplitude	(no) (no)	challenging trivial