

# HORIZONTAL-VERTICAL EMITTANCE EXCHANGE SCHEMES IN A RING

**P. Raimondi, J. Chavanne**

*ESRF - ASD*

**A. Chao (SLAC)**

*SLAC*

- **New generation storage rings have very small horizontal emittance (although not quite diffraction limited) and can still reach very high  $E_x/E_y$  emittance ratio thanks to well consolidated technologies, diagnostic and tuning techniques**
- **Typically the vertical emittance is well below the diffraction limit**
- **Potentially an overall increase of the brightness could be obtained by trading between the two planes**
- **The simplest method is by coupling the two planes having a final emittance  $E_x/2$  in both planes. This solution is very attractive when  $E_x$  is already very close to the diffraction limit or for specific cases needed by the Users**
- **The emittance exchanger locally generates round beams with emittances about  $\sqrt{E_x \cdot E_y}$ . However the beam is not matched to the undulator. The matching is improved for low energy rings, strongest solenoids and long undulators.**
- **Another efficient method could be a trade off of the vertical and horizontal partition numbers. A significative reduction of the horizontal emittance (or longitudinal...) while still maintaing the vertical at the diffraction level seems possible.**

- **This emittance exchange within a solenoid has been extensively studied in the past few years.**
- **Theory is well established**

- [1] Y. Derbenev, Michigan Univ. Report No. 91-2 (1991); UM HE 93-20 (1993); Workshop on Round Beams and Related Concepts in Beam Dynamics, Fermilab (1996).
- [2] Y. Derbenev, Michigan Univ. Report UM HE 98-04 (1998).
- [3] A. Burov and V. Danilov, FNAL Report No. TM-2040 (1998); FNAL Report TM-2043 (1998).
- [4] R. Brinkmann, Y. Derbenev, and K. Floettmann, Phys. Rev. Special Topics – Accel. & Beams, Vol. 4, 053501 (2001).
- [5] A. Burov, S. Nagaitsev, and Ysroslov Derbenev, Phys. Rev. E 66, 016503 (2002).
- [6] A. Burov, S. Nagaitsev, A. Shemyakin, and Ya. Derbenev, Phys. Rev. Special Topics – Accel. & Beams, Vol. 3, 094002 (2000).
- [7] Kwang-Je Kim, Phys. Rev. Special Topics – Accel. & Beams, Vol. 6, 104002 (2003)
- [8] D. Edwards et al., Proc. 2001 Part. Accel. Conf., Chicago, IL (IEEE, Piscataway, NJ, 2001), p. 73.
- [9] P. Piot, Y.-E. Sun, K.-J. Kim, Phys. Rev. Special Topics – Accel. & Beams, Vol. 9, 031001 (2006).
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- [11] E.D. Courant, H.S. Snyder, Ann. Phys. 3, 1 (1958).

- **Several experiments have been successfully made, e.g: to go from round to flat emittances**

## **THE FLAT BEAM EXPERIMENT AT THE FNAL PHOTOINJECTOR**

D. Edwards, H. Edwards, N. Holtkamp, S. Nagaitsev, J. Santucci, FNAL\*  
R. Brinkmann, K. Desler, K. Flöttmann, DESY-Hamburg  
I. Bohnet, DESY-Zeuthen, M. Ferrario, INFN-Frascati

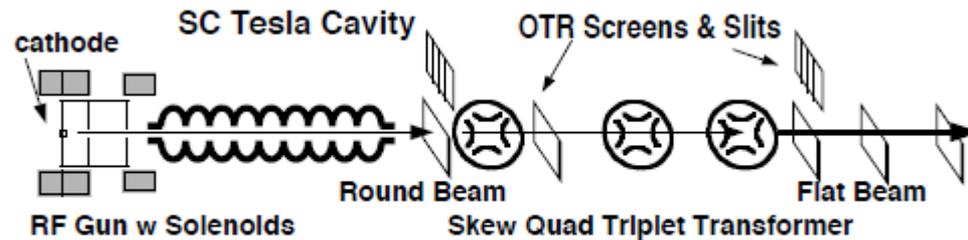


Figure 1: Very schematic rendition of the layout at Fermilab related to this experiment.

## Flat Beam generation from a round cathode

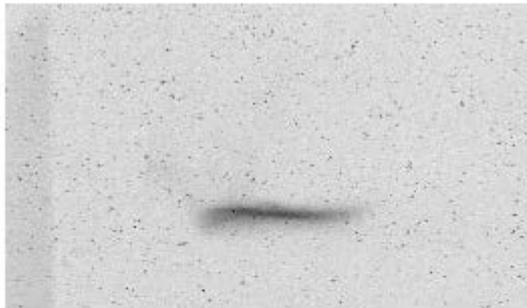


Figure 2: Beam profile on OTR screen 1.2 m downstream of the third skew quadrupole.

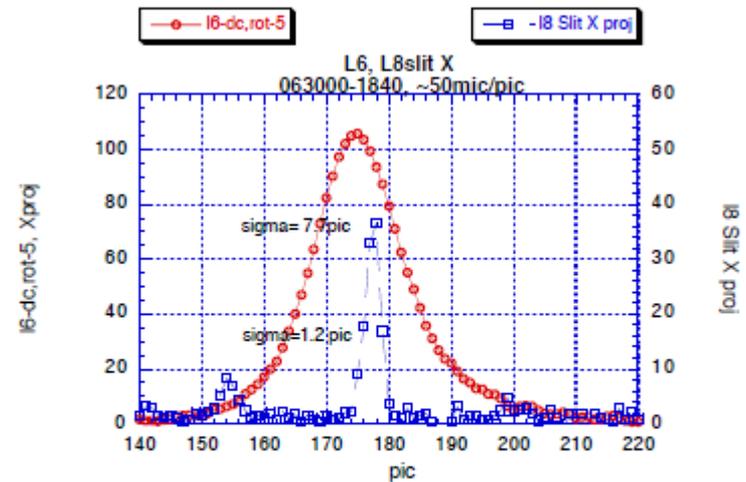


Figure 4: Projection of images used in emittance measurement at slit location and downstream of slit system.

## Reminder of the principle....(idealized)

We can track a parallel beam  $X_{in}$  from inside the solenoid through the its exit.

Then we have a triplet with 90deg phase advance difference in the two planes.

We obtain a flat beam  $X_{out}$  tilted 45deg w.r.t. the normal axis.

$$X_{in} = \begin{pmatrix} x_0 \\ 0 \\ y_0 \\ 0 \end{pmatrix}; \quad Sol_{exit} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k_s/2 & 0 \\ 0 & 0 & 1 & 0 \\ k_s/2 & 0 & 0 & 1 \end{bmatrix}$$

$$Triplet = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & 1/\beta & 0 \end{bmatrix}; \quad \beta = 2/k_s; \quad X_{out} = \begin{pmatrix} x_0 \\ -k_s/2 \cdot y_0 \\ x_0 \\ -k_s/2 \cdot y_0 \end{pmatrix}$$

Changing the triplet in a skew (45deg tilt) channel, will generate a flat beam:

$$X_{flat} = \begin{pmatrix} \sqrt{2} \cdot x_0 \\ -k_s/\sqrt{2} \cdot y_0 \\ 0 \\ 0 \end{pmatrix}$$

# Application in a ring....

We start from a beam with  $\beta_x = \beta_y$  and  $\alpha_x = \alpha_y = 0$ :

$$\Sigma_0 = \begin{bmatrix} \beta_x \varepsilon_x & 0 & 0 & 0 \\ 0 & \varepsilon_x / \beta_x & 0 & 0 \\ 0 & 0 & \beta_x \varepsilon_y & 0 \\ 0 & 0 & 0 & \varepsilon_y / \beta_x \end{bmatrix}$$

Then we have a triplet (skewed) with :  
(V with triplet not rotated)

$$V = \begin{bmatrix} \cos(\varphi + \frac{\pi}{4}) & \beta_x \sin(\varphi + \frac{\pi}{4}) & 0 & 0 \\ -\frac{1}{\beta_x} \sin(\varphi + \frac{\pi}{4}) & \cos(\varphi + \frac{\pi}{4}) & 0 & 0 \\ 0 & 0 & \cos(\varphi - \frac{\pi}{4}) & \beta_x \sin(\varphi - \frac{\pi}{4}) \\ 0 & 0 & -\frac{1}{\beta_x} \sin(\varphi - \frac{\pi}{4}) & \cos(\varphi - \frac{\pi}{4}) \end{bmatrix}$$

Then a solenoid with  $K_s = 2/\beta_x$ .

Inside the solenoid we will have:

$$\Sigma_{sol} = \begin{bmatrix} (\varepsilon_x + \varepsilon_y) / k_s & 0 & 0 & \varepsilon_y \\ 0 & \varepsilon_y k_s & -\varepsilon_y & 0 \\ 0 & -\varepsilon_y & (\varepsilon_x + \varepsilon_y) / k_s & 0 \\ \varepsilon_y & 0 & 0 & \varepsilon_y k_s \end{bmatrix}$$

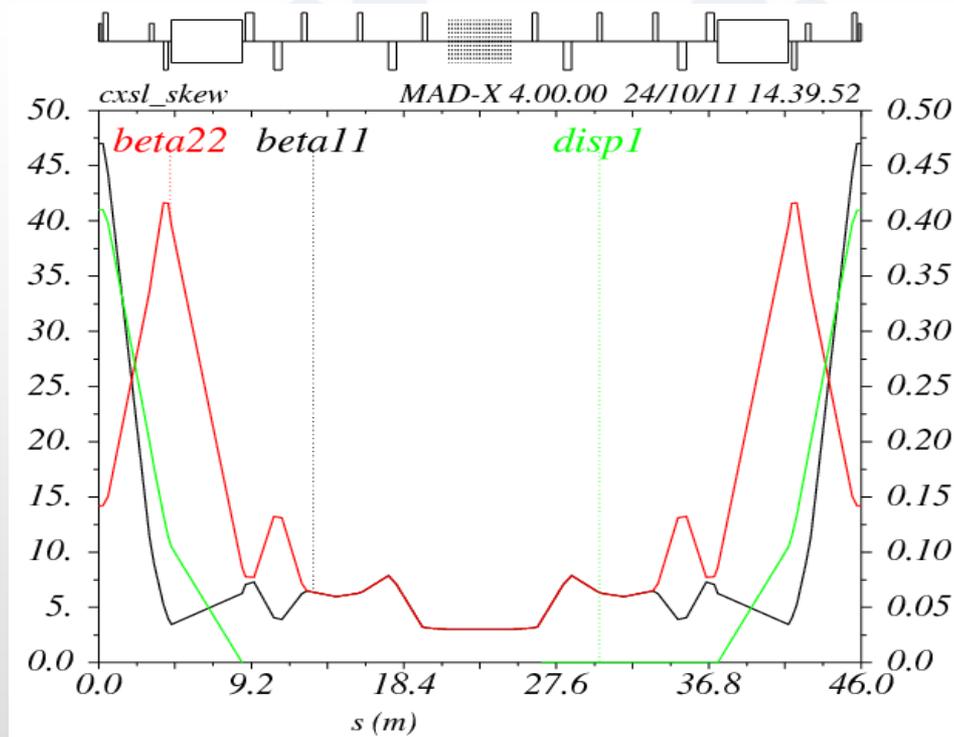
After the solenoid there is an identical triplet and the beam returns uncoupled

We have to remark that the x and y emittances in the canonical coordinates  $(x, p_x, y, p_y)$  are larger. They are the average of the two, as shown by the Sigma matrix expressed in canonical coordinates:

$$\Sigma_{can} = \begin{bmatrix} \frac{\varepsilon_x + \varepsilon_y}{k_s} & 0 & 0 & -\frac{1}{2}(\varepsilon_x - \varepsilon_y) \\ 0 & \frac{k_s}{4}(\varepsilon_x + \varepsilon_y) & \frac{1}{2}(\varepsilon_x - \varepsilon_y) & 0 \\ 0 & \frac{1}{2}(\varepsilon_x - \varepsilon_y) & \frac{\varepsilon_x + \varepsilon_y}{k_s} & 0 \\ -\frac{1}{2}(\varepsilon_x - \varepsilon_y) & 0 & 0 & \frac{k_s}{4}(\varepsilon_x + \varepsilon_y) \end{bmatrix}$$

Fortunately the beam properties of interest are determined by the mechanical momenta, so the effective X and Y emittances in the solenoid are roughly:

$$\sqrt{\varepsilon_x \varepsilon_y}$$



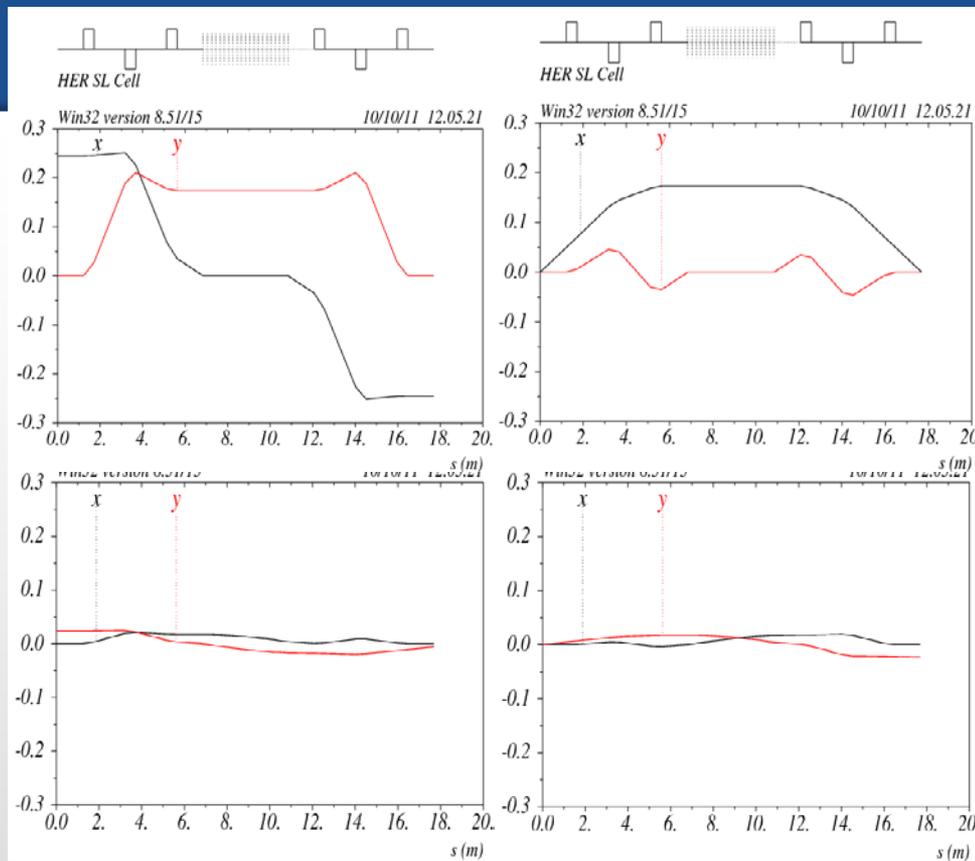
- An example of an insertion. After the Dispersion suppressor a triplet makes  $\beta_x = \beta_y = 6\text{m}$  and  $\alpha_x = \alpha_y = 0$  at the entrance of the emittance adapter.
- The adapter consists of the solenoid and two skew-triplet on each side. The solenoid has:  $k_s = 2/\beta_x = 0.333/\text{m}$  (4.2T for a 6GeV beam)
- K values for the skew-quads (50cm long) are around 1

Skew triplet transport matrix (in the upright configuration) is:

$$V = \begin{bmatrix} \cos(\varphi + \frac{\pi}{4}) & \beta_x \sin(\varphi + \frac{\pi}{4}) & 0 & 0 \\ -\frac{1}{\beta_x} \sin(\varphi + \frac{\pi}{4}) & \cos(\varphi + \frac{\pi}{4}) & 0 & 0 \\ 0 & 0 & \cos(\varphi - \frac{\pi}{4}) & \beta_x \sin(\varphi - \frac{\pi}{4}) \\ 0 & 0 & -\frac{1}{\beta_x} \sin(\varphi - \frac{\pi}{4}) & \cos(\varphi - \frac{\pi}{4}) \end{bmatrix}$$

We have set  $\varphi = \pi/2$  as an example, but its value is arbitrary. In general the incoming parameters  $\beta$  and  $\alpha$  in the adapter section are arbitrary as well, provided that they are equal in the two planes.

For lower values of  $\varphi$  the quads become weaker and the section less chromatic.



Tracking through the adapter for incoming beam displacements ( $\varepsilon_x = 10 \text{ nm}$ ,  $\varepsilon_y = 10^{-2} \varepsilon_x$ ):

$$x = 10^{-4} \sqrt{\beta_x}; x' = 10^{-4} / \sqrt{\beta_x}; y = 10^{-5} \sqrt{\beta_x}; y' = 10^{-5} / \sqrt{\beta_x}$$

For this particular phase  $x_{in}$  becomes pure  $y$  in the solenoid and  $x'_{in}$  pure  $x$ .

The vertical emittance contributes just to the  $\sigma_{x'}$  and  $\sigma_{y'}$  in the solenoid

## Match the beam with the undulator:

- The beam angular divergences in the undulator are:

$$\sigma_{x'} = \sigma_{y'} = \sqrt{\varepsilon_y k_s}$$

For  $\varepsilon_y = 3\text{pm}$  and  $k_s = 0.333$  it correspond to about 1urad!

In order not to spoil the potential gain in brightness we need:

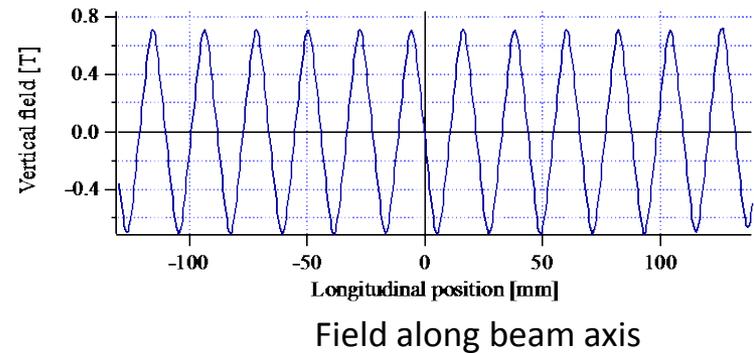
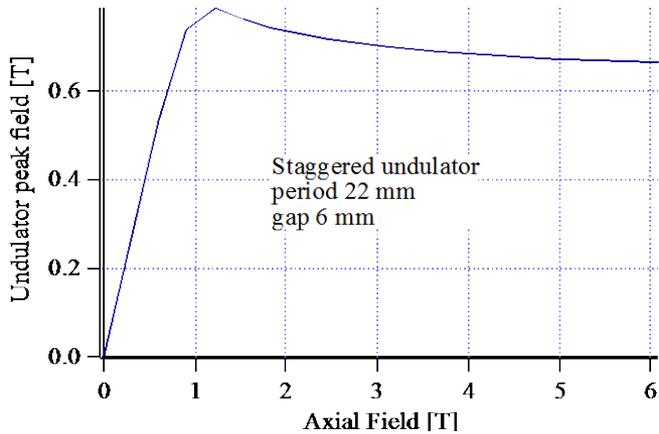
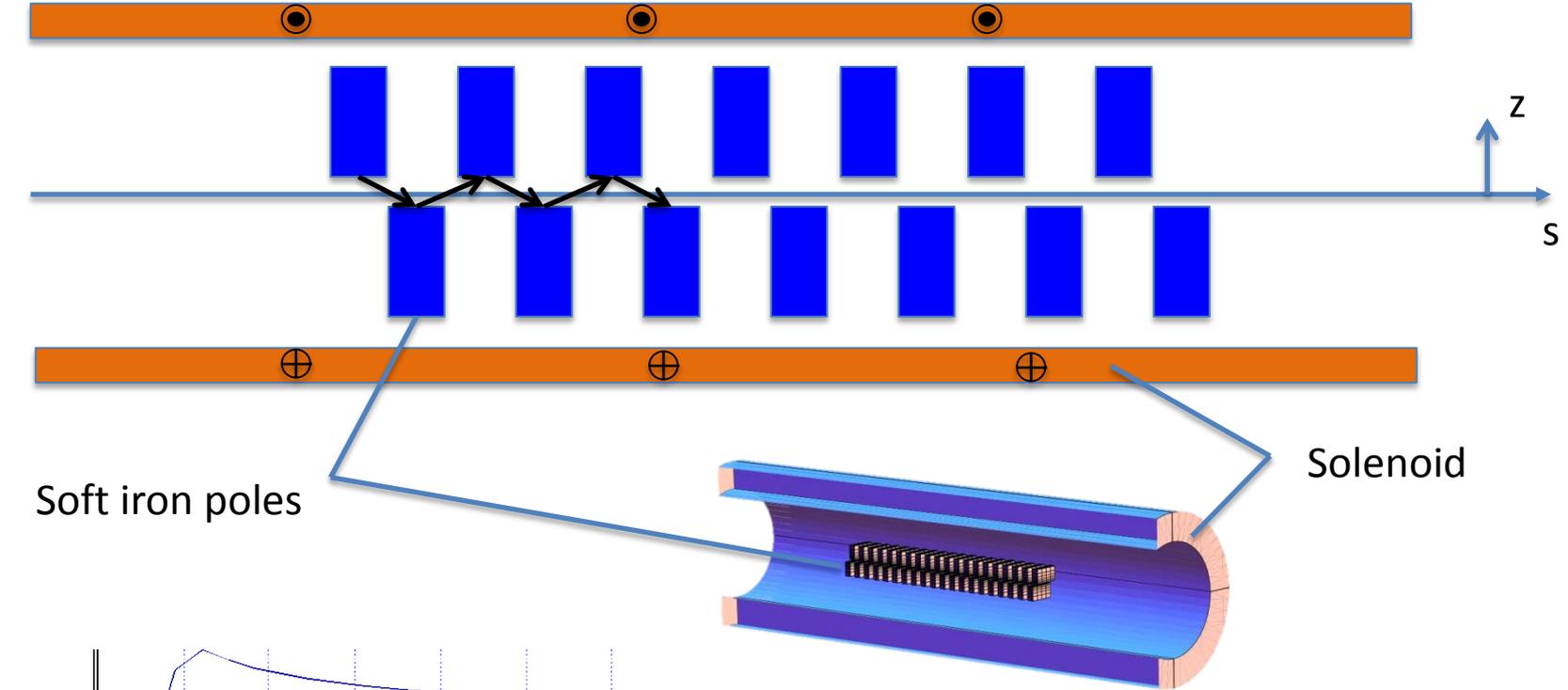
- 1) the undulator to be as long as possible ( $L > 5\text{-}10\text{m}$ )
- 2) the solenoid as strong as possible ( $k_s > 1$ ,  $B > 12\text{T}$  @  $6\text{GeV}$ )

**These requirements are very hard to get.**

**It is not going to be easy to find a suitable solution for the existing rings.**

**The key element is the realization of an Undulator in a very powerful SC Solenoid with very large k...**

# Possible undulator scheme: Staggered undulator **J Chavanne**



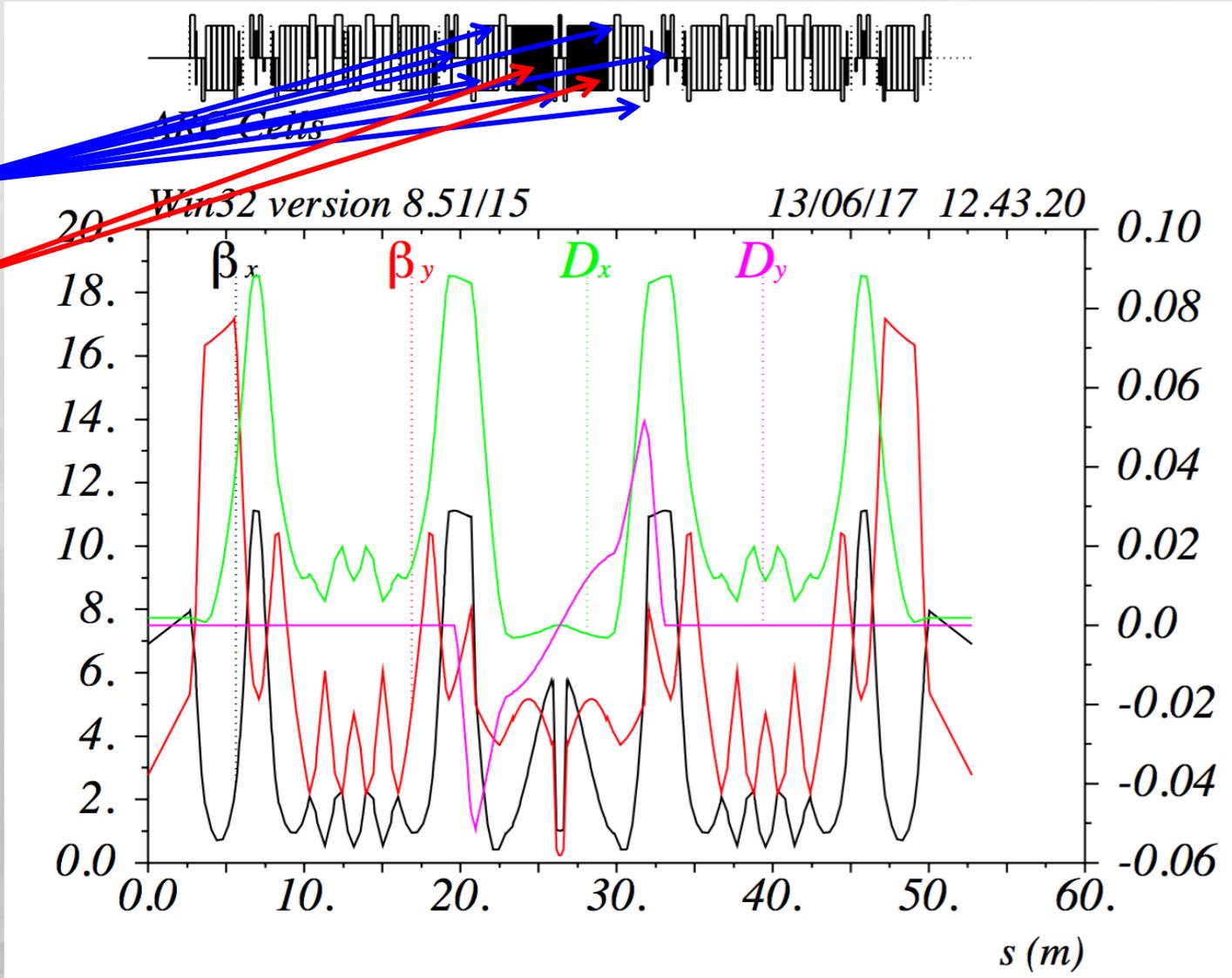
Trade off of vertical and horizontal partition numbers could be done by means of powerful Robinson Wigglers (RWs) placed in a region where there is vertical (and horizontal) dispersion.

However in order to generate a sizable vertical dispersion a vertical chicane (or better a dogleg) is needed. The dipoles have to be weak and the optics well optimized to not create too much vertical emittance.

Another possibility is to rotate the beam locally as for the emittance exchanger case:

- In the area where the beam is somewhat rotated the dipoles and the RWs will be effectively acting on the vertical plane.
- The geometry of the ring is unchanged
- By varying vertical and horizontal dispersion in the RWs with matching quads  $J_y$ ,  $J_x$  and  $J_z$  can be mutually exchanged

4\*2 Skew  
Quads  
2 Robinson  
Wigglers



**The beam rotation (coupling) is made with 4\*2 skew quads around and inside the straight section**

**The Beam is coupled only in the straight section**

**Two Robinson Wigglers 2.5m long with a gradient of about 300T/m are inserted in the straight section**

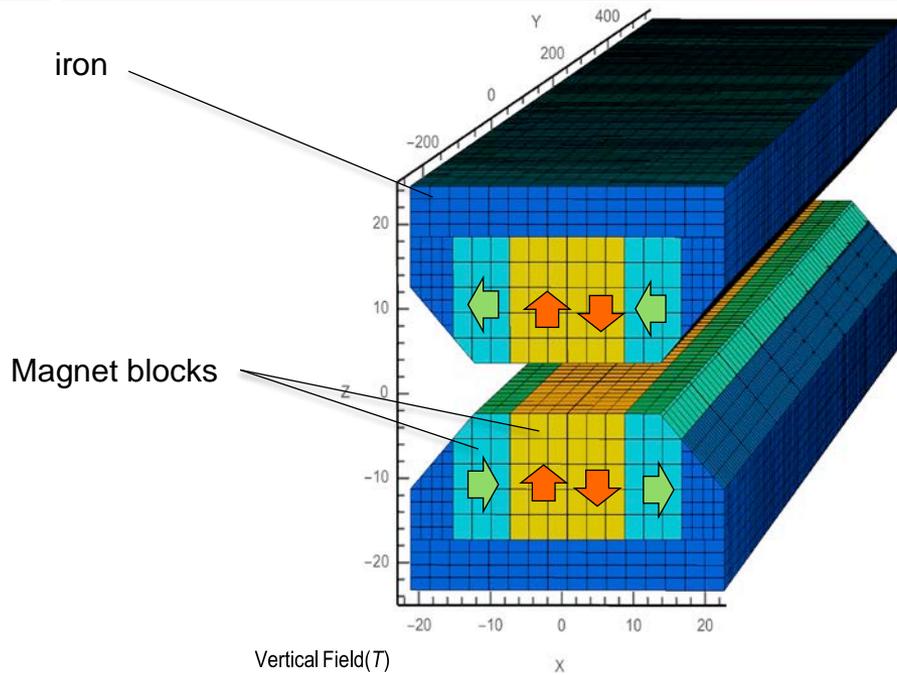
**$J_y$  is reduced to 0.3**

**$J_x$  is increased from 1.5 to 2.2**

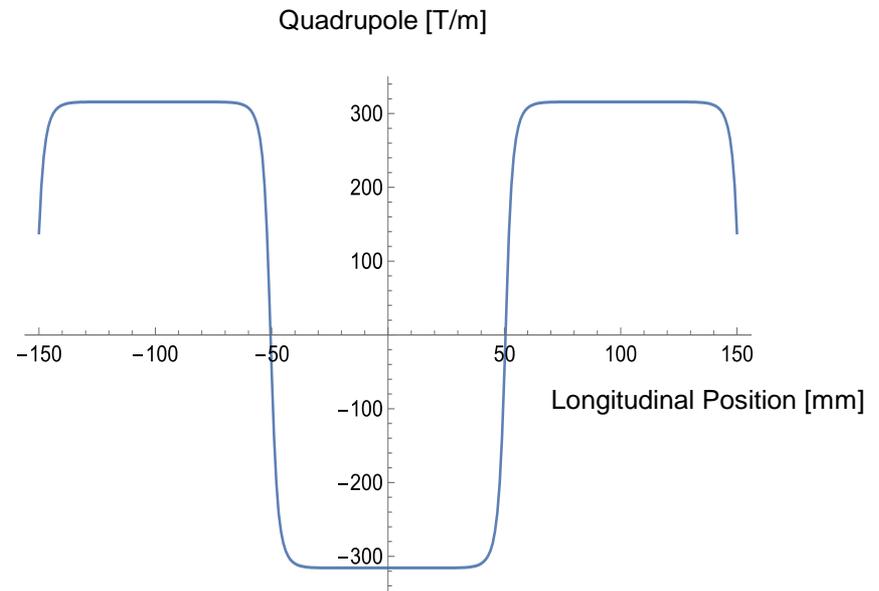
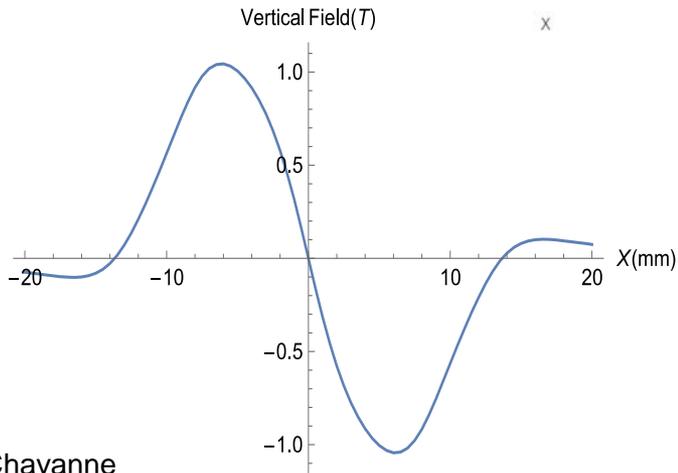
**The vertical emittance due to the insertion is about 4pm**

**The horizontal emittance is reduced from 133pm to about 90pm**

# QUADRUPOLE UNDULATOR J CHAVANNE



Period 100mm  
Gap 6 mm  
Peak gradient  $\sim 300$  T/m  
Permanent magnet  $B_r : 1.45$  T



J. Chavanne

- The emittance exchanger has the potential to lower the horizontal emittance in the Insertion Devices.
- The optics requirements for the transport line are easy to meet with the present technology.
- The requirements for the Undulator+Solenoid are very challenging.
  
- Jx-Jy trade off could be an intermediate (and tunable) step toward round beams
  - Potentially almost the same gain in the horizontal emittance obtained by coupling the ring  $E_x \Rightarrow E_x/2$  while keeping a very small vertical emittance
  - The requirements for the RWs are very challenging but similar to the state of the art present InVac undulators.