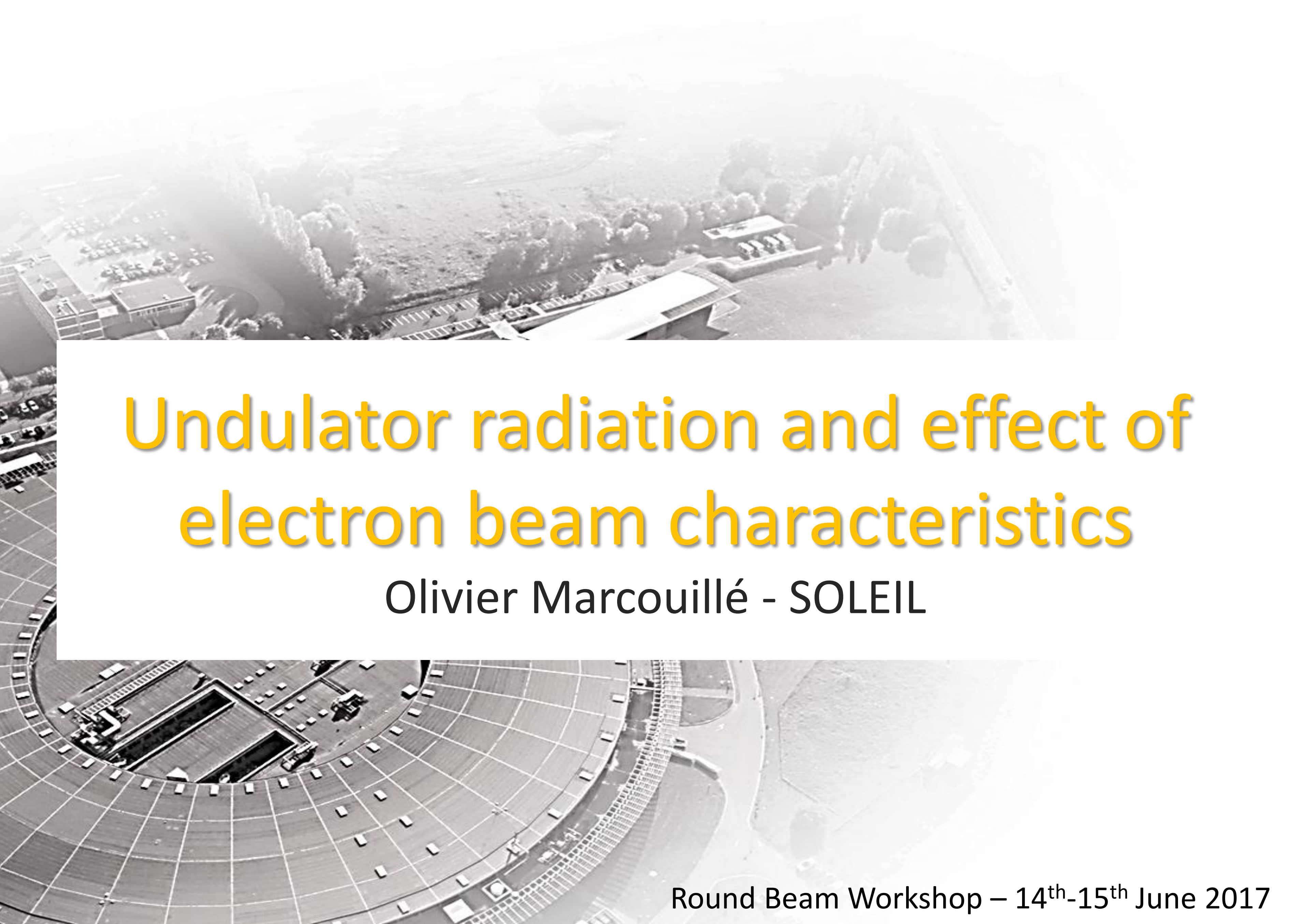


Undulator radiation and effect of electron beam characteristics

Olivier Marcouillé - SOLEIL

Round Beam Workshop – 14th-15th June 2017

Introduction

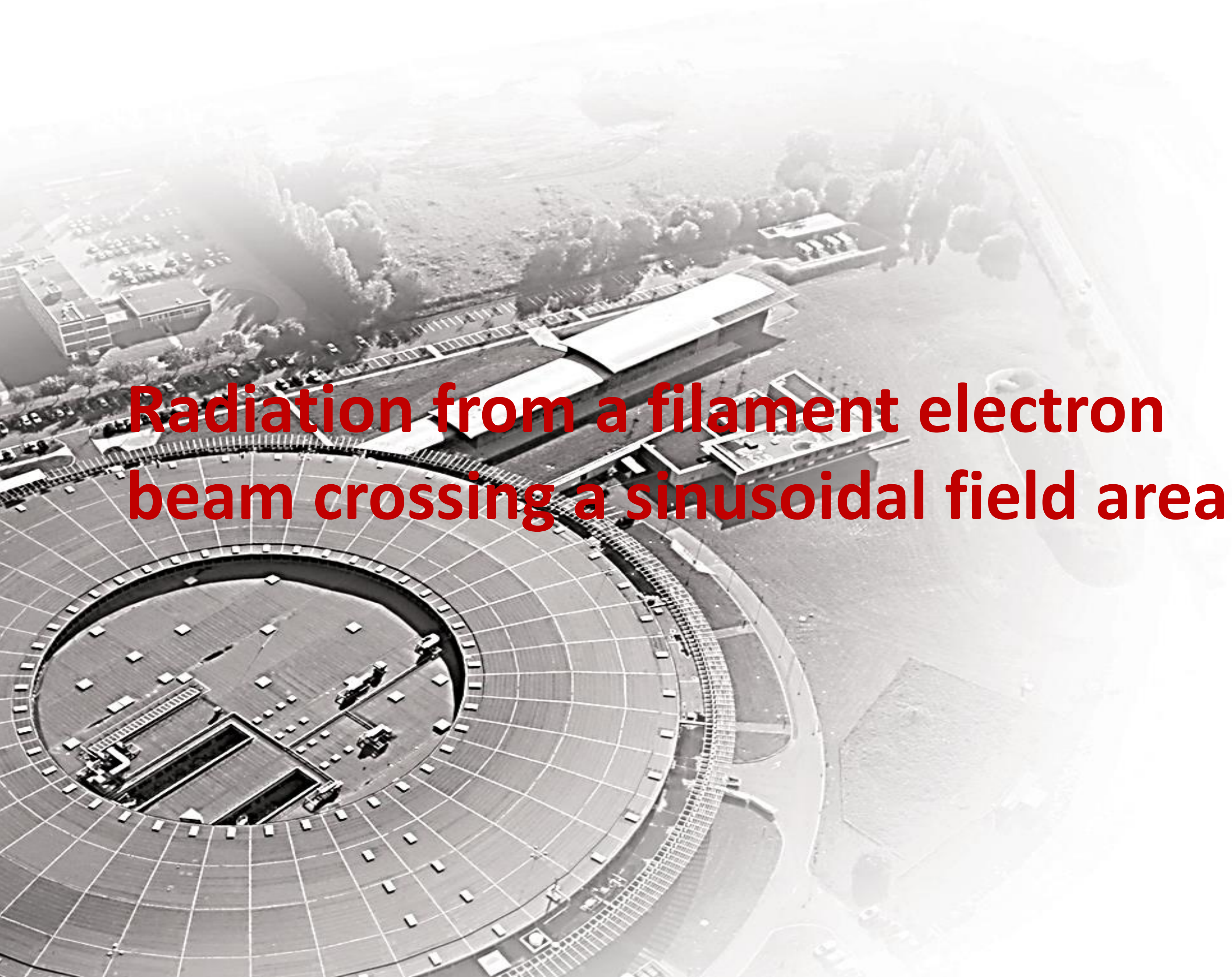
The present talk aims at presenting the characteristics of the radiation emitted by electron beam crossing a planar undulator in terms of spectral flux and bandwith. We will see that the spectrum emitted by a monoenergetic divergence free electron beam presents a series of lines with thickness of the order of 10^{-4} and even less. However, how do the optical performances and also the bandwith evolves if the electron beam owns an energy spread and also an angular divergence and position? How is the spectrum changed if the electron beam motion is disturbed when crossing the undulator?

I will try to point out these effects to estimate the impact on the optical performances of a planar undulator owning a longitudinal field component on its propagation axis

Contents

- **Radiation from a filament electron beam**
  - Motion of electrons in a planar sinusoïdal undulator
  - Electric field and spectrum
  - Source size
  - Spectral flux
- **Effects of beam characteristics**
  - Emittance
  - Energy spread
- **Changes in the electron beam motion**
  - Magnetic imperfections
  - Adding a longitudinal Field component
- **Summary and conclusions**

Radiation from a filament electron beam crossing a sinusoidal field area

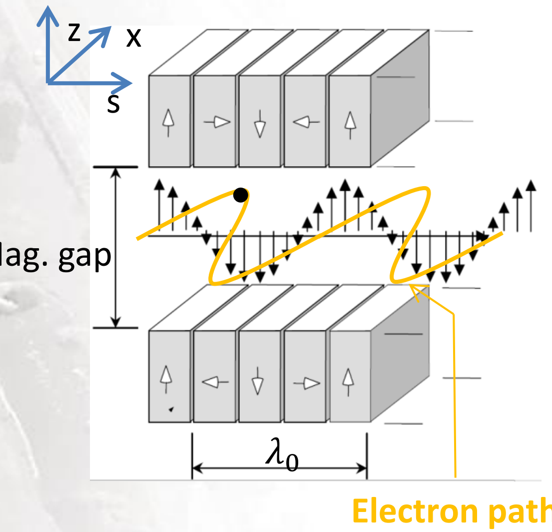


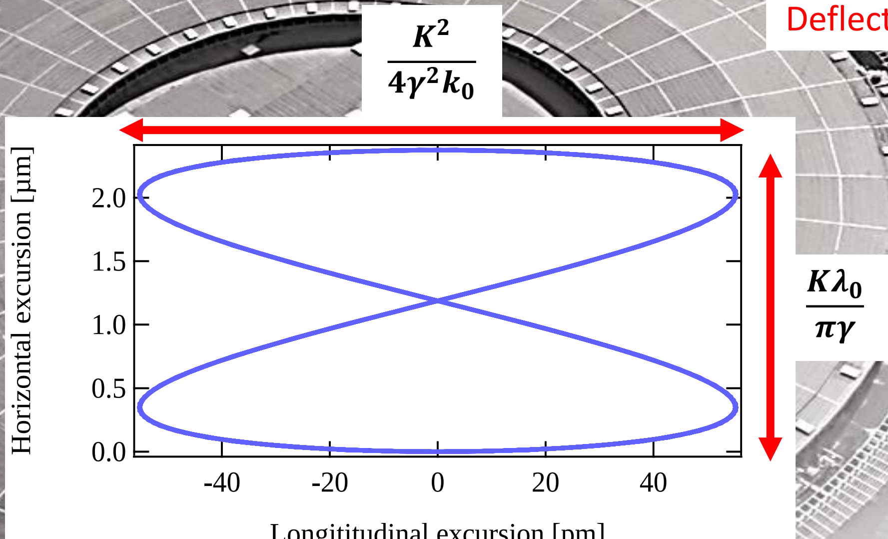
Motion of electrons in a planar sinusoïdal undulator

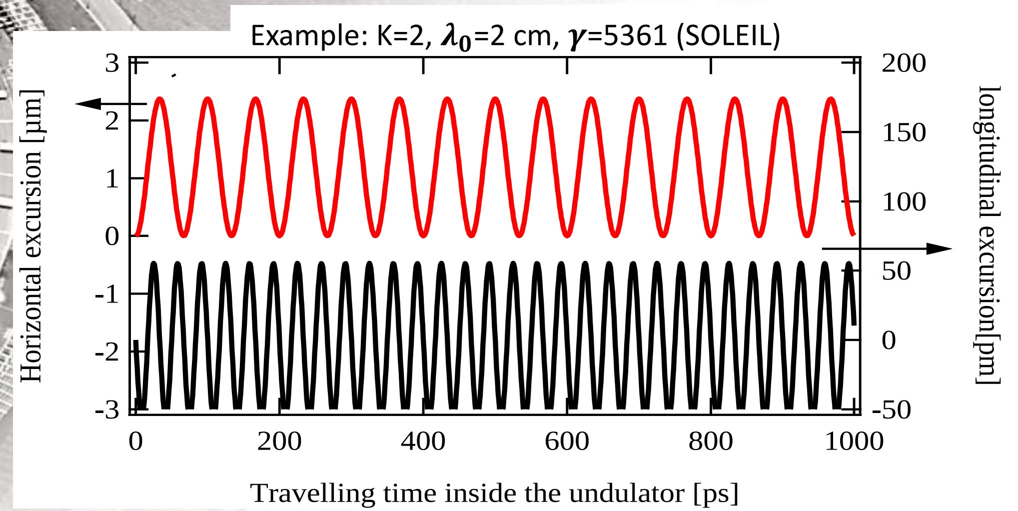
The electron motion is ruled by the Lorentz equation: $\gamma m_0 \frac{d\vec{v}}{dt} = -e\vec{v} \times \vec{B}$. In the case of a planar undulator : $\vec{B}(z) = B_0 \cos(k_0 s)\vec{s}$. The normalized speed $\vec{\beta} = \frac{\vec{v}}{c}$ is written as:

- $\boldsymbol{\beta_x = \frac{K}{\gamma}\sin(k_0 ct), \beta_z=0, \beta_s \sim 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2}{2}\right] - \frac{K^2}{4\gamma^2}\cos(2k_0 ct)}$
- $\boldsymbol{x \cong \frac{K\lambda_0}{2\pi\gamma}[1 - \cos(k_0 ct)]}$
- $\boldsymbol{z \cong \overline{\beta_s} ct - \frac{K^2}{8\gamma^2 k_0}\sin(2k_0 ct)}$ with $K = \frac{eB_0\lambda_0}{2\pi m_0 c} = 0.934 B_0[T]\lambda_0[cm]$

Deflection parameter





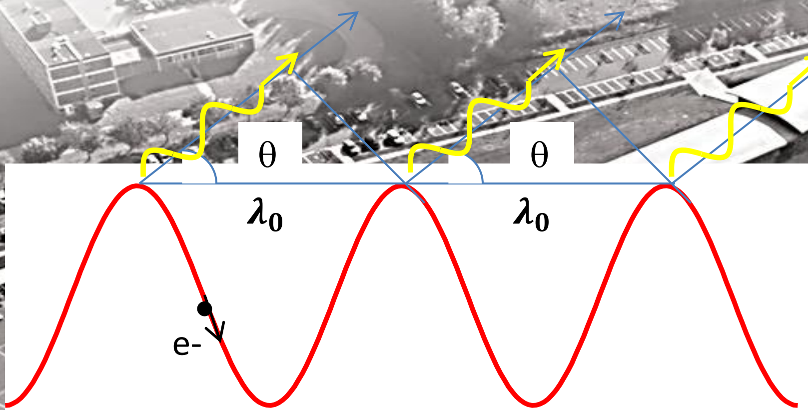


Resonant Energy

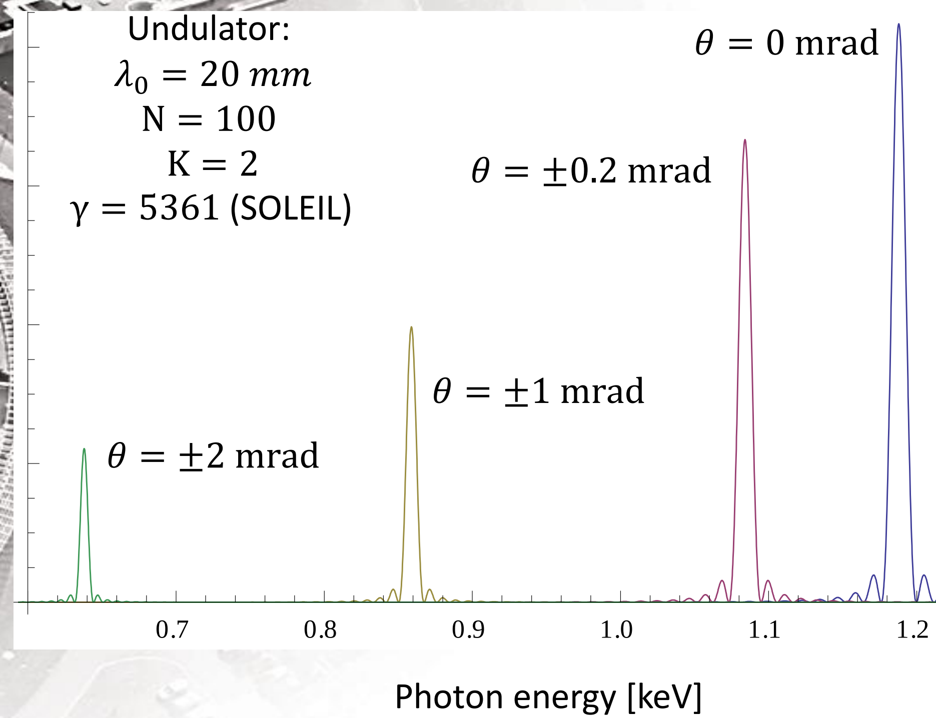
$$E_r(\theta) = \frac{0.95\, {E_e}^2[GeV]}{\lambda_0[mm]\left[1+\frac{K^2}{2}+\gamma^2\theta^2\right]} = \frac{h}{2\pi}\omega_r(\theta)$$

In keV

γ: Lorentz factor
λ_0: magnetic period
θ: Observation Angle

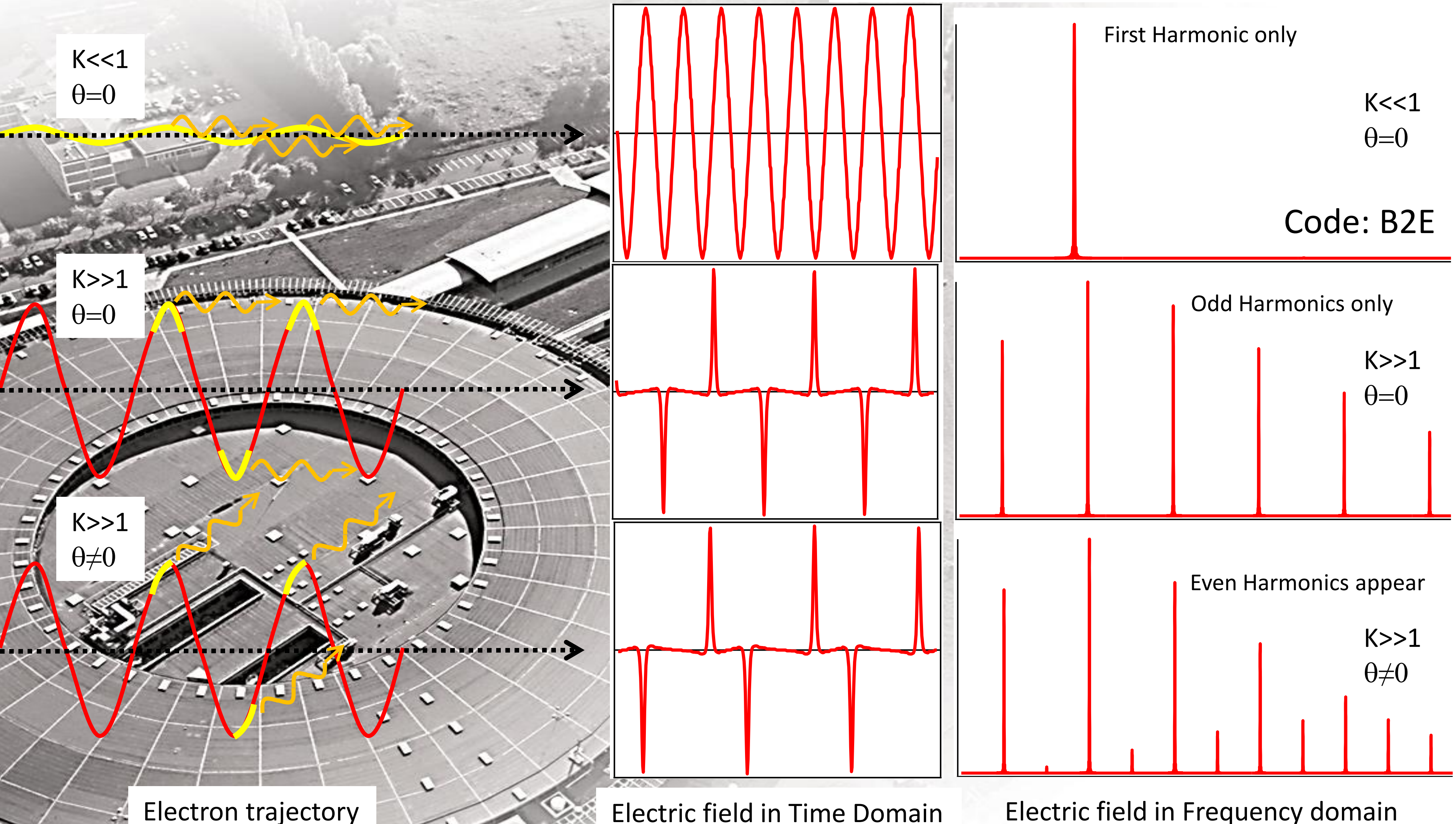


$$\lambda_r(\theta) = \frac{\lambda_0}{2n\gamma^2}\left[1+\frac{K^2}{2}+\gamma^2\theta^2\right]$$

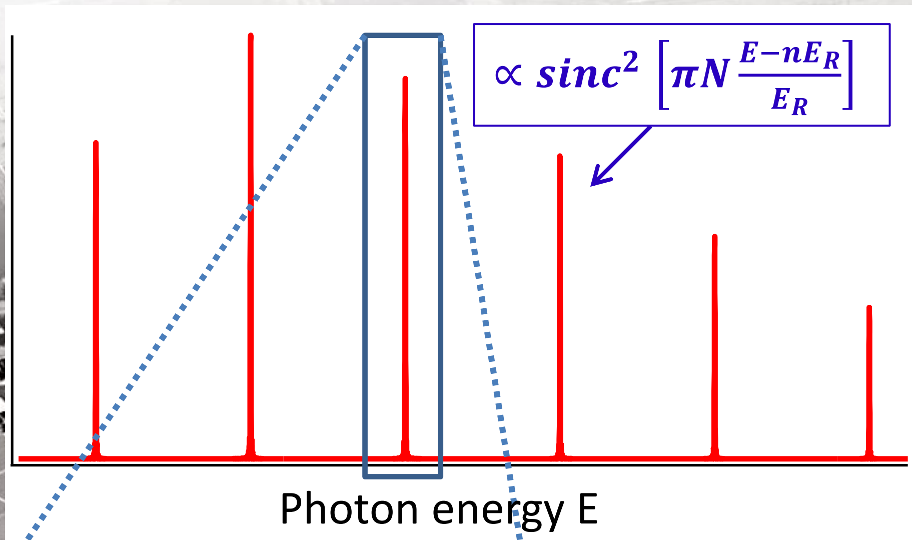


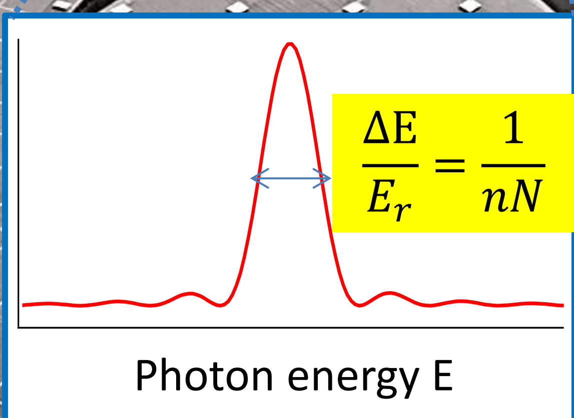
Radiated electric field

$$\overrightarrow{E(\omega)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \vec{E}(t) e^{-j\omega t} dt$$



Natural bandwith, angular divergence and source size





n: harmonic number
N: period number

N.A.:
n=19-21, N=100, ΔE/E=5 10^{-4}

$$\lambda_r(\theta) = \frac{\lambda_0}{2n\gamma^2}\left[1 + \frac{K^2}{2} + \gamma^2\theta^2\right]$$

Emission angle $\sigma_\nu' = \theta_2 - \theta_1$ such as:

$$\frac{\Delta E}{E_r} = \frac{\Delta\lambda}{\lambda_r} = \frac{E_r(\theta_2) - E_r(\theta_1)}{E_r} = \frac{1}{nN}$$

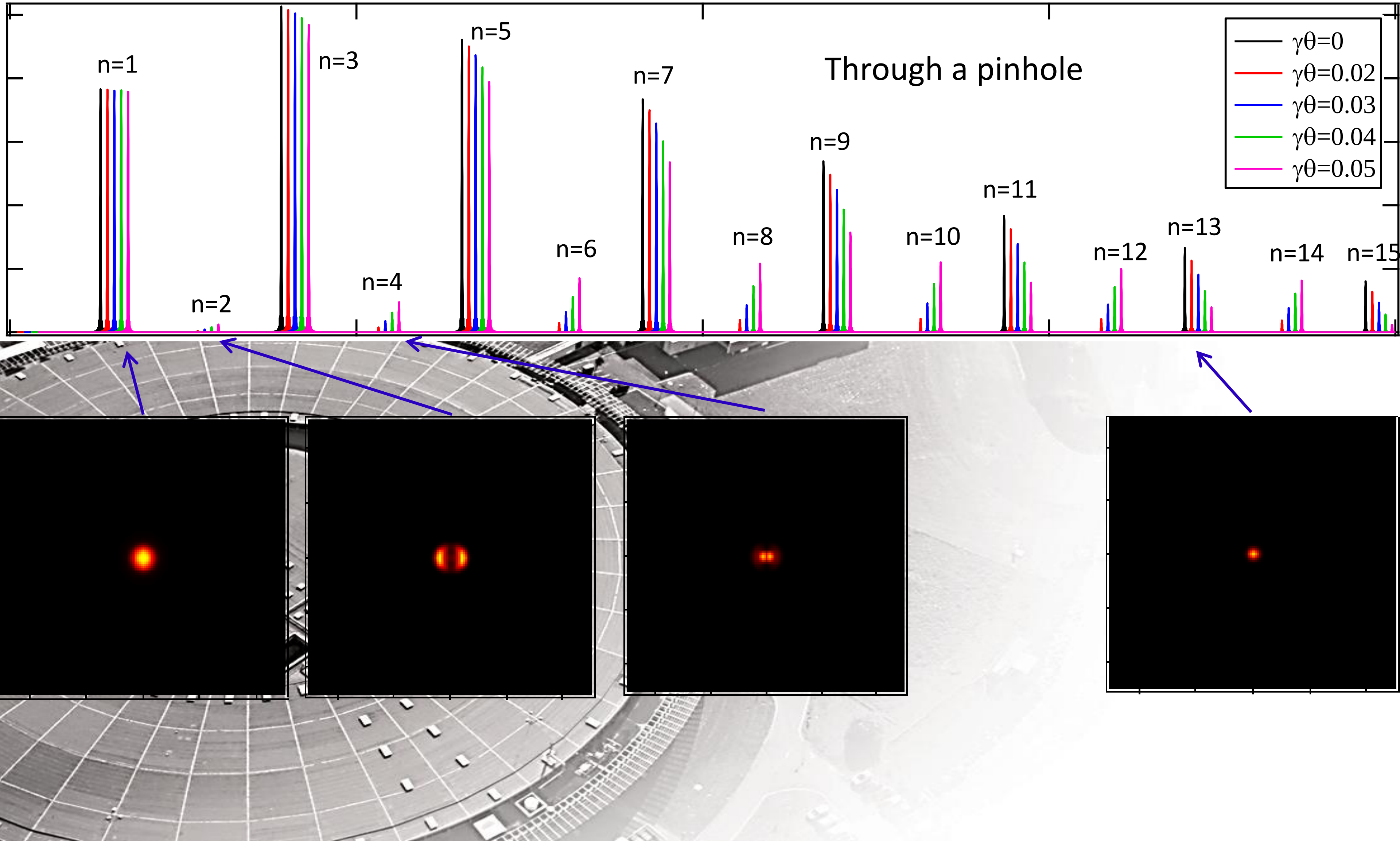
Photon divergence

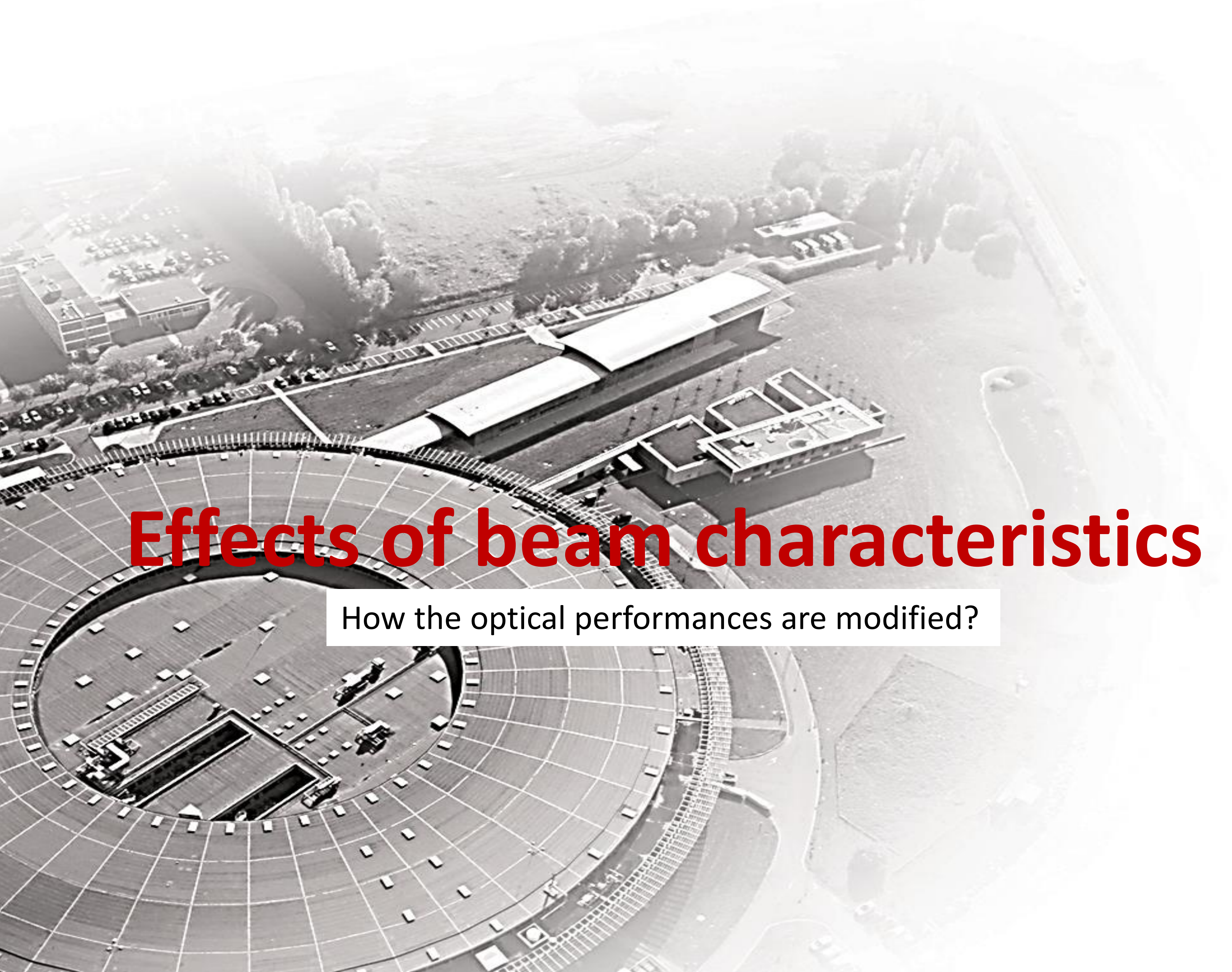
$\sigma_\nu' = \sqrt{\frac{\lambda}{2L}}$ at resonance

Photon emittance (gaussian beam): $\sigma_\nu \sigma_\nu' \sim \frac{\lambda}{4\pi}$

Photon size $\sigma_\nu \sim \frac{\sqrt{2\lambda L}}{4\pi}$

Spatial distribution





Effects of beam characteristics

How the optical performances are modified?

Twiss parameters

The electron beam optics of a synchrotron facility is defined by the **Twiss parameters**:

- The emittance in horizontal and vertical plane: respectively ε_x and ε_z
- The betatron functions in horizontal and vertical plane: respectively β_x and β_z
- The dispersion functions in horizontal and vertical plane: respectively η_x and η_z

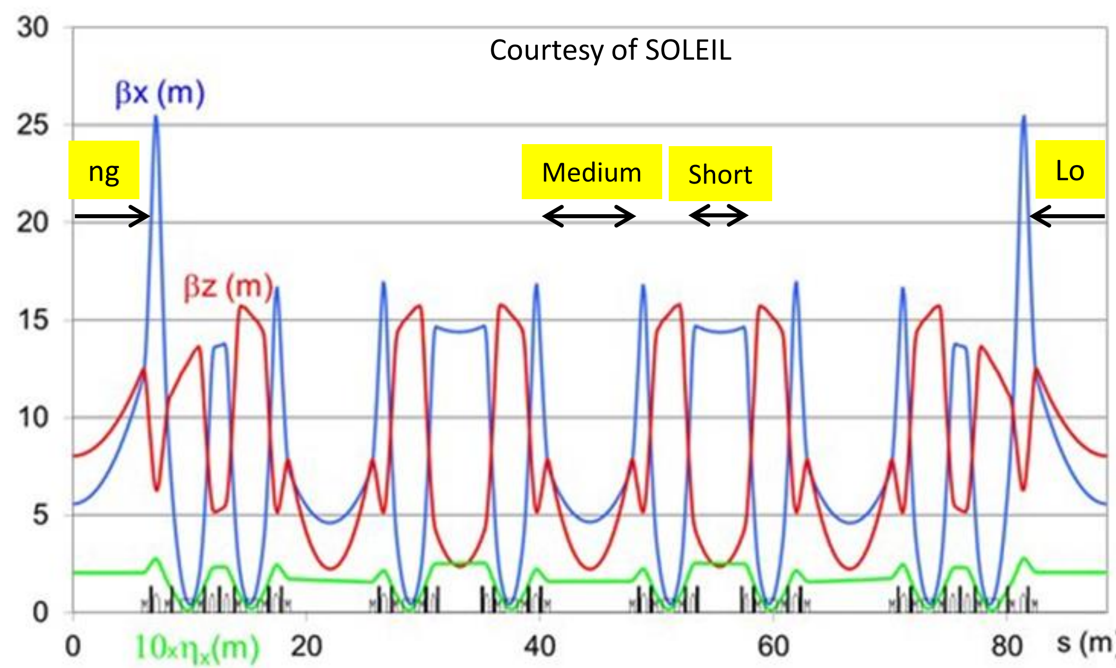
And also the relative energy spread σ_E/E or σ_γ/γ.

The Twiss parameters determines the electron beam **size and divergence** at the undulator location:

- $\sigma_x = \sqrt{\varepsilon_x \beta_x + {\eta_x}^2 \left(\frac{\sigma_\gamma}{\gamma}\right)^2}$
- $\sigma_z = \varepsilon_z \beta_z$
- $\sigma_x' = \sqrt{\frac{\varepsilon_x}{\beta_x}\left(1 + \frac{{\beta_x}'^2}{4}\right) + {\eta_x}'^2 \left(\frac{\sigma_\gamma}{\gamma}\right)^2}$
- $\sigma_z' = \sqrt{\frac{\varepsilon_z}{\beta_z}\left(1 + \frac{{\beta_z}'^2}{4}\right)}$

At the Straight section center:

$\beta_x' = \beta_z' = \eta_x' = \eta_z' = 0$



Photon source size and angular divergence

- Source size: $\Sigma_{x,z} = \sqrt{\sigma_{x,z}{}^2 + \sigma_\nu{}^2}$ → $\sigma_\nu \sim \frac{\sqrt{\lambda L}}{4\pi}$

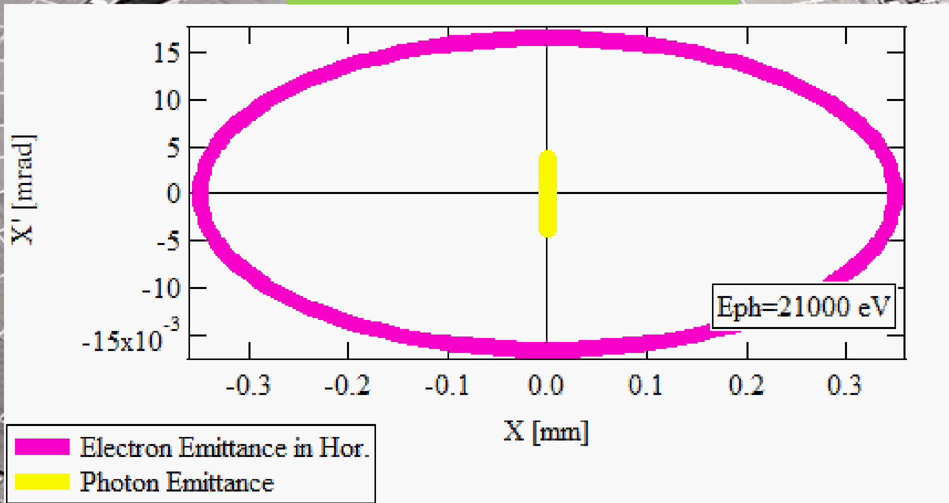
Electron beam contribution | Photon beam contribution

- Angular divergence: $\Sigma_{x,z}{}' = \sqrt{\sigma_{x\prime,z\prime}{}^2 + \sigma_\nu{}'^2}$ → $\sigma_\nu{}' = \sqrt{\frac{\lambda}{2L}}$

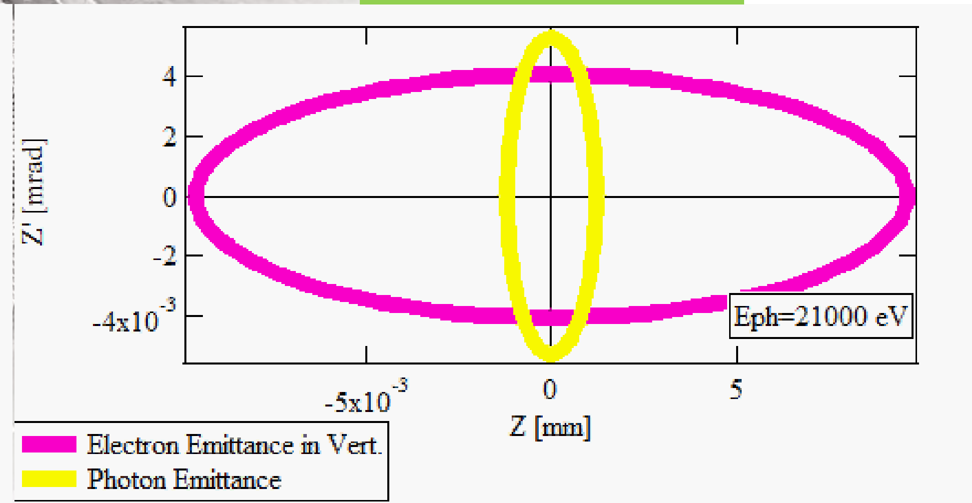
- Diffraction limited when : $\boldsymbol{\sigma_{x,z} < \sigma_\nu}$ **and** $\boldsymbol{\sigma_{x\prime,z\prime} < \sigma_\nu{}'}$.

U20 :Period of 20 mm operating between 1000 eV and 10000 eV

HORIZONTAL ELLIPSE



VERTICAL ELLIPSE



Spectral broadening

$$\lambda_r(\theta) = \frac{\lambda_0}{2n\gamma^2}\left[1 + \frac{K^2}{2} + \gamma^2\theta^2\right]$$

The spectral bandwith results from the homogeneous broadening and from the inhomogeneous broadening:

$$\left[\frac{\Delta\lambda}{\lambda}\right]_{Total} = \frac{1}{nN} + \left[\frac{\Delta\lambda}{\lambda}\right]_{\sigma_\gamma} + \left[\frac{\Delta\lambda}{\lambda}\right]_{\sigma_x{}'} + \left[\frac{\Delta\lambda}{\lambda}\right]_{\sigma_z{}'}$$

- Homogeneous broadening:

$$\left[\frac{\Delta\lambda}{\lambda}\right]_{Hom} = \frac{1}{nN}$$

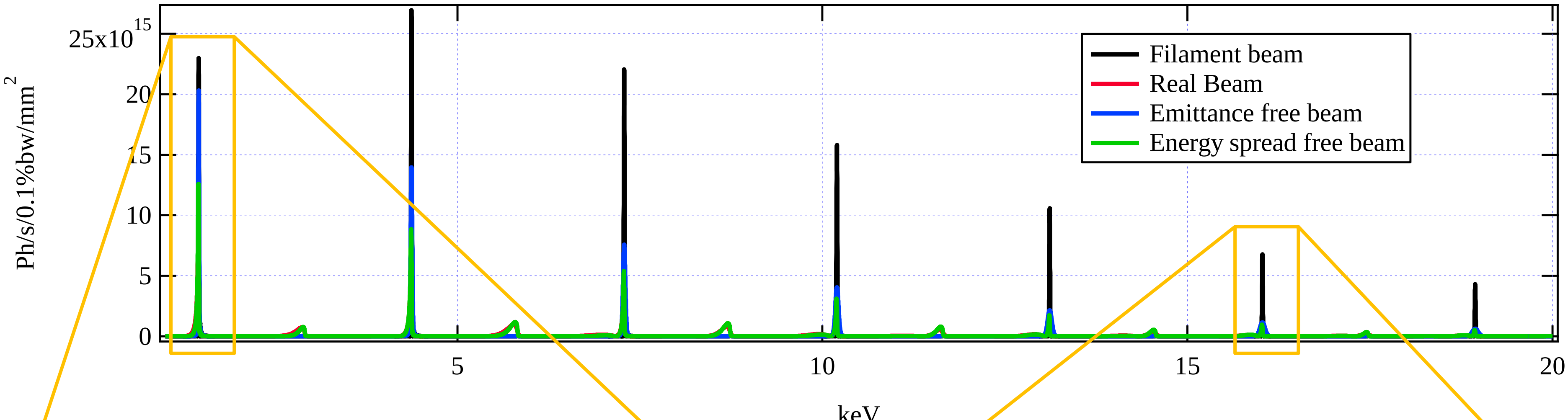
- Inhomogeneous broadening
  - Energy spread σ_γ/γ:

$$\left[\frac{\Delta\lambda}{\lambda}\right]_{\sigma_\gamma} = 2\frac{\sigma_\gamma}{\gamma}$$

  - Angular divergence $\sigma_x{}'$ et $\sigma_z{}'$:

$$\left[\frac{\Delta\lambda}{\lambda}\right]_{\sigma_{x,z}{}'} = \frac{\gamma^2\sigma_{x\prime,z\prime}{}^2}{1 + K^2/2}$$

Spectral distribution: emittance and energy spread

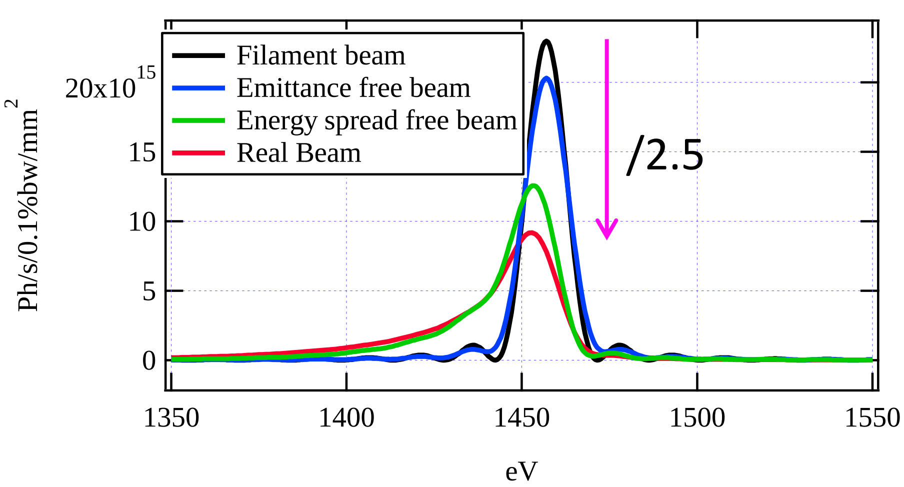


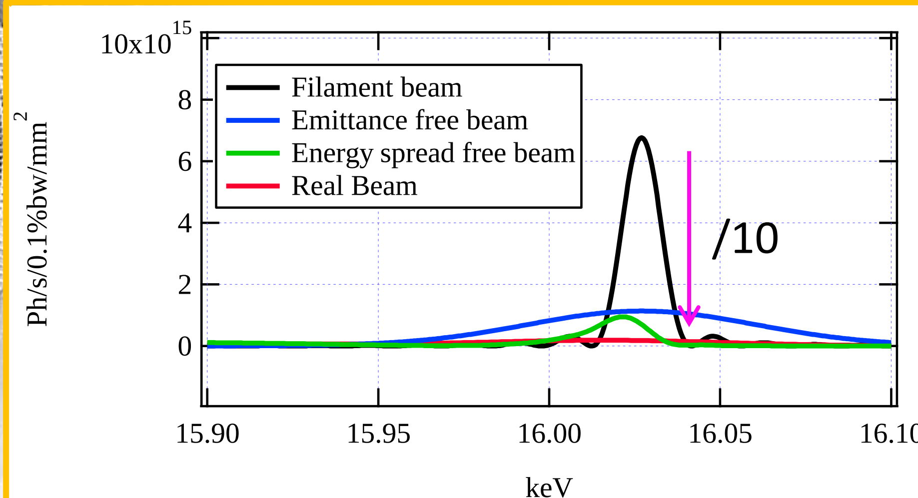
Filament: $\varepsilon_x = \varepsilon_z = \frac{\sigma_\gamma}{\gamma} = 0$

Emittance free: $\varepsilon_x = \varepsilon_z = 0$

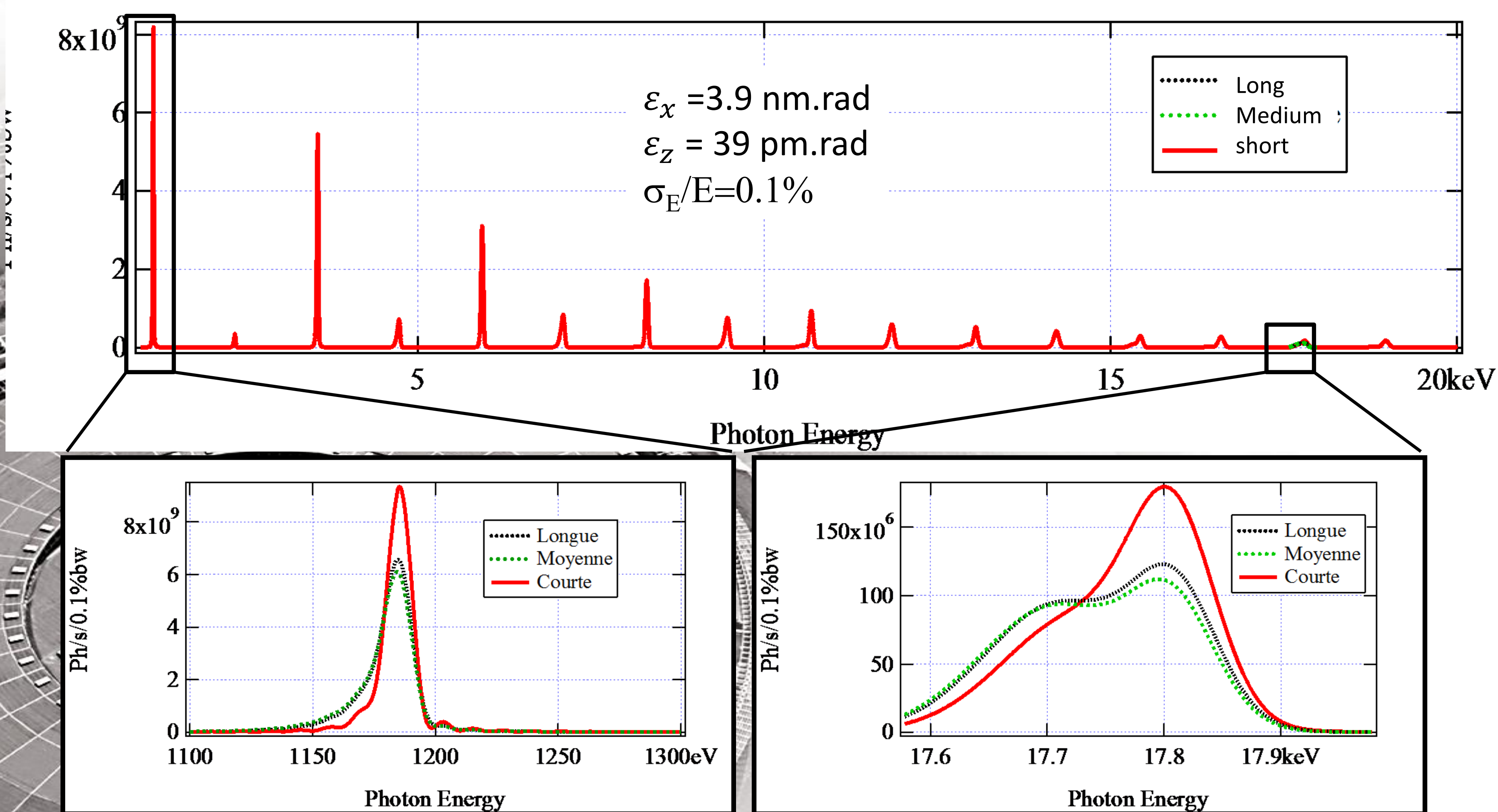
Energy spread: $\frac{\sigma_\gamma}{\gamma} = 0$

Real: ε_x **= 3.9 nm.rad** ε_z **= 39 pm.rad** $\frac{\sigma_\gamma}{\gamma} = 0.001$

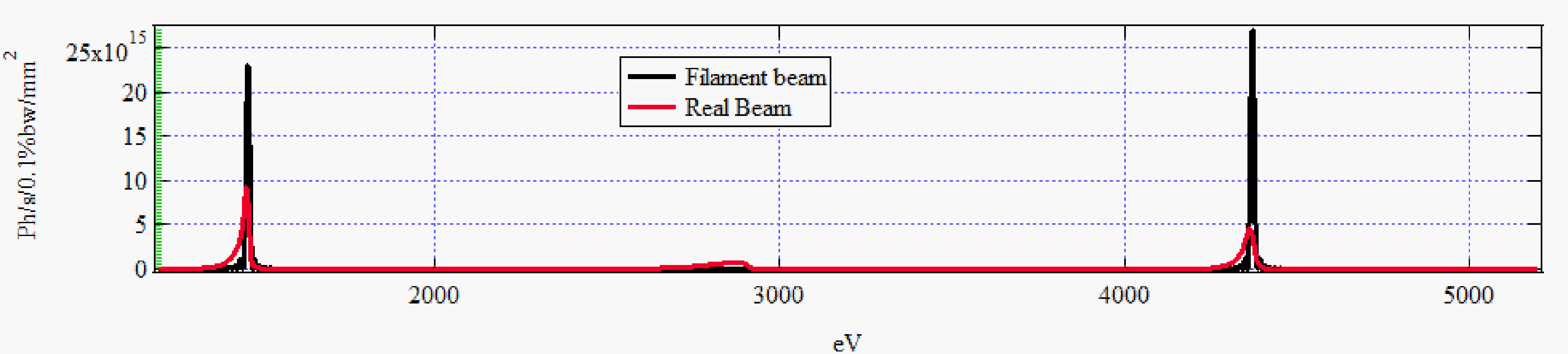




Spectral distribution: beta function



Spatial distribution

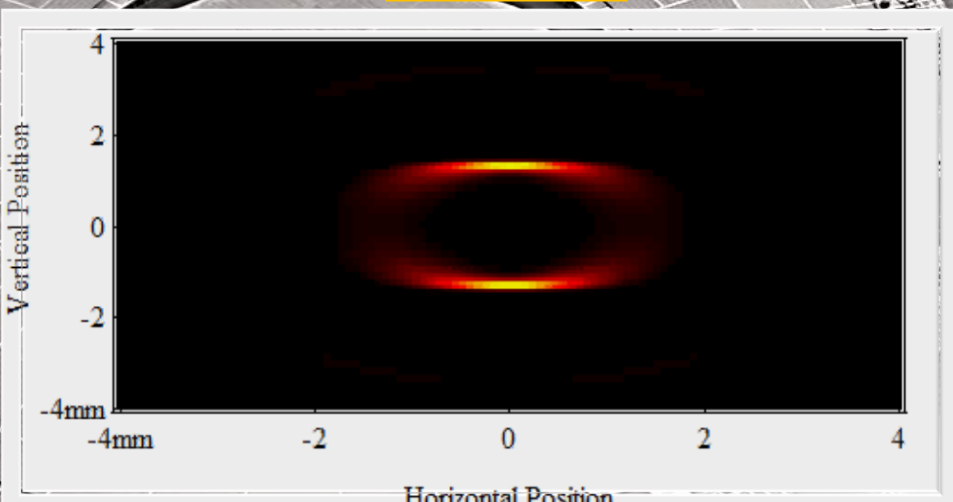


@10 m from the undulator center

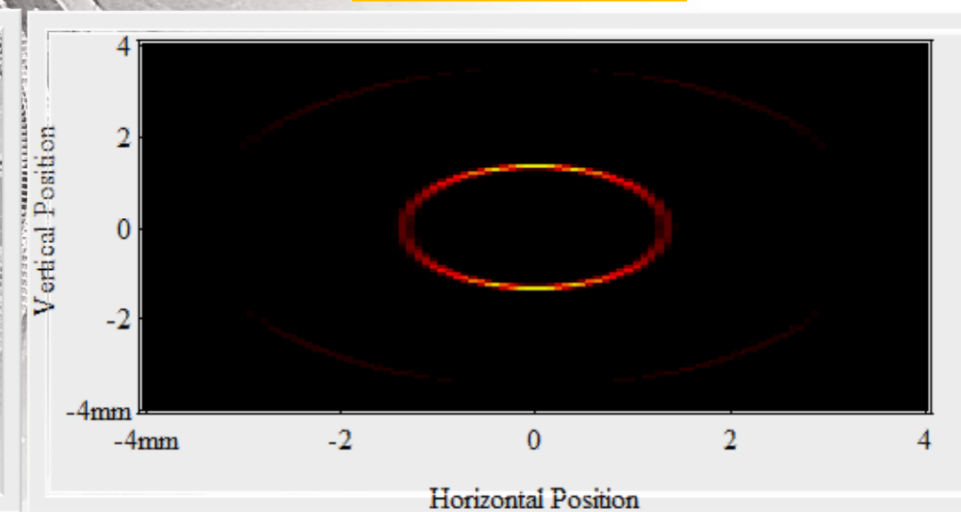
1200 eV ⟷ 5200 eV

Eph=1200 eV

REAL BEAM



FILAMENT BEAM



Changes in the electron beam motion

Magnetic defaults

$x \cong \frac{K\lambda_0}{2\pi\gamma}[1 - cos(k_0 ct)]$

- Local δK result in trajectory magnitude variation $\frac{\delta K\lambda_0}{2\pi\gamma}$

→Increase of the source size

- Local δK result in phase error increase coming

From the non steady regime of photon burst

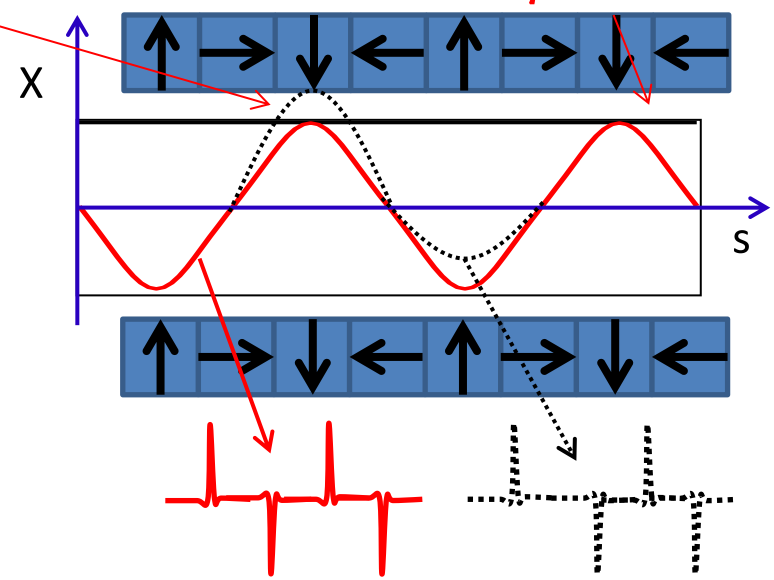
Emission.

$\overrightarrow{E_T} = \sum_k^{2N} \overrightarrow{E_k} e^{j\varphi_k} \cong \overrightarrow{E_T} = \overrightarrow{E_0} \sum_k^{2N} e^{j\varphi_k}$

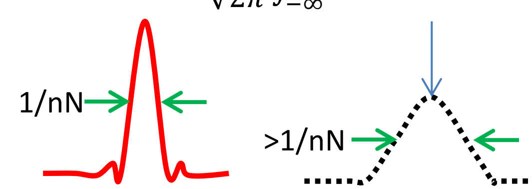
σ_ϕ is the standard deviation of φ_k (gaussian distribution).

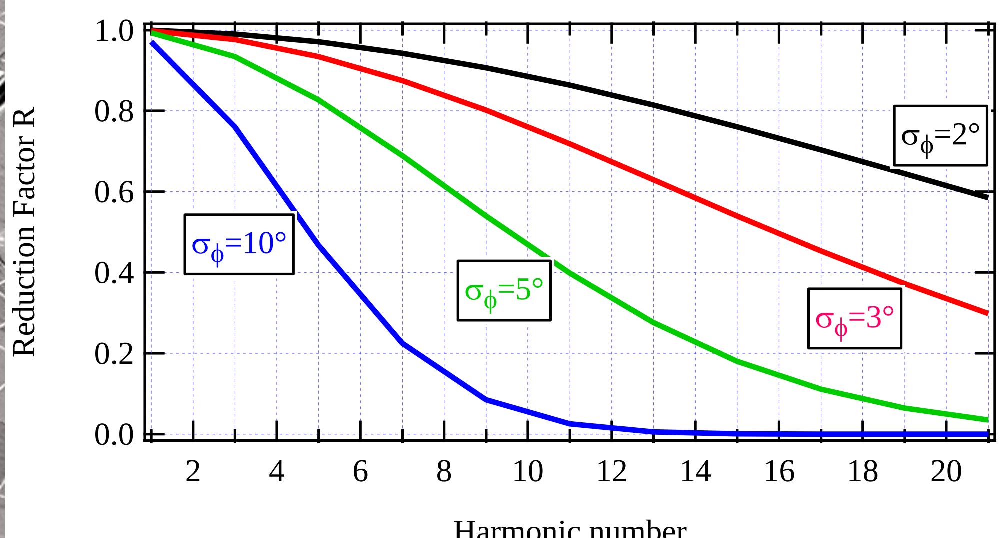
One can estimate the reduction factor of the radiation intensity:

$R = exp[-n^2 {\sigma_\phi}^2]$



$\overrightarrow{E(\omega)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \vec{E}(t) e^{-j\omega t} dt$



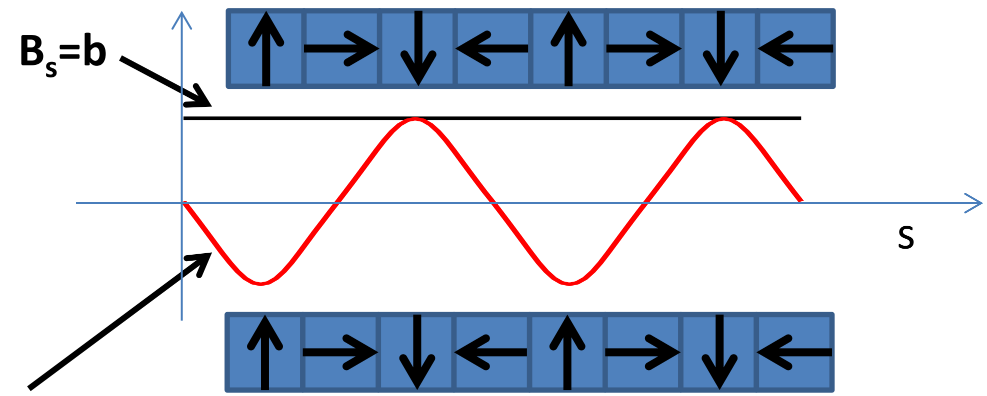


- Increase of the bandwidth
- Reduction of intensity

Adding a longitudinal field component (1/6)

A longitudinal component B_s is superimposed to the vertical periodic field. What will be the motion of particule and how will the emission be changed. The motion respects the analytical relations:

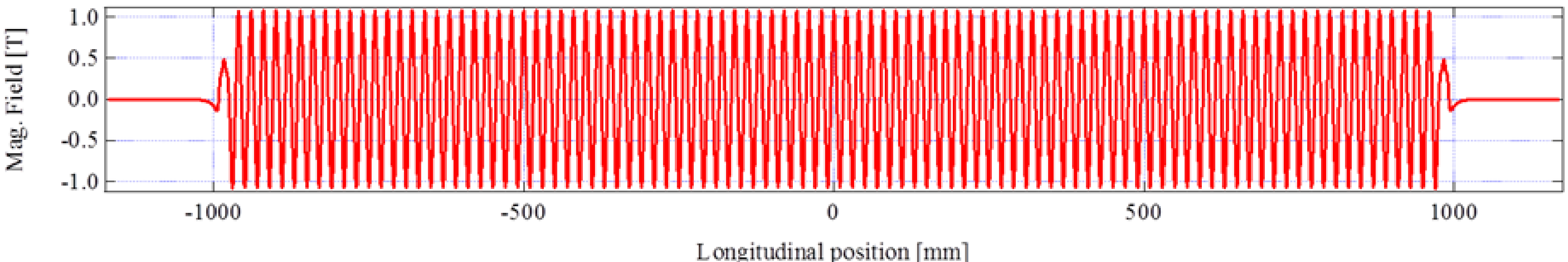
- $\frac{d\beta_x}{ds} = \frac{-e}{\gamma m_0 c}(\beta_z B_s - \beta_s B_z)$
- $\frac{d\beta_z}{ds} = \frac{e}{\gamma m_0 c}\beta_x B_s$

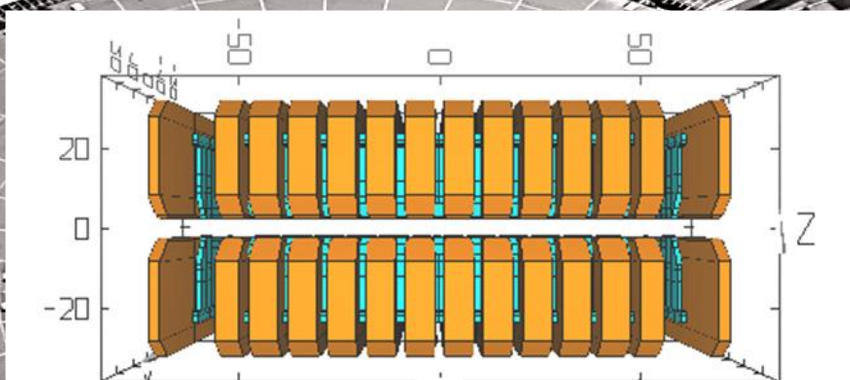


$B_z(s) = B_0 \cos(k_0 s)$

β_x and β_z are calculated by Runge-Kutta integration taking into account the initial conditions at the entrance of the undulator (β_{x0} and β_{z0}). The positions X and Z are also deduced by simple integration of the normalized speed and taking into account the initial conditions at the entrance of the undulator (X_0 and Z_0).

Adding a longitudinal field component (2/6): magnetic field



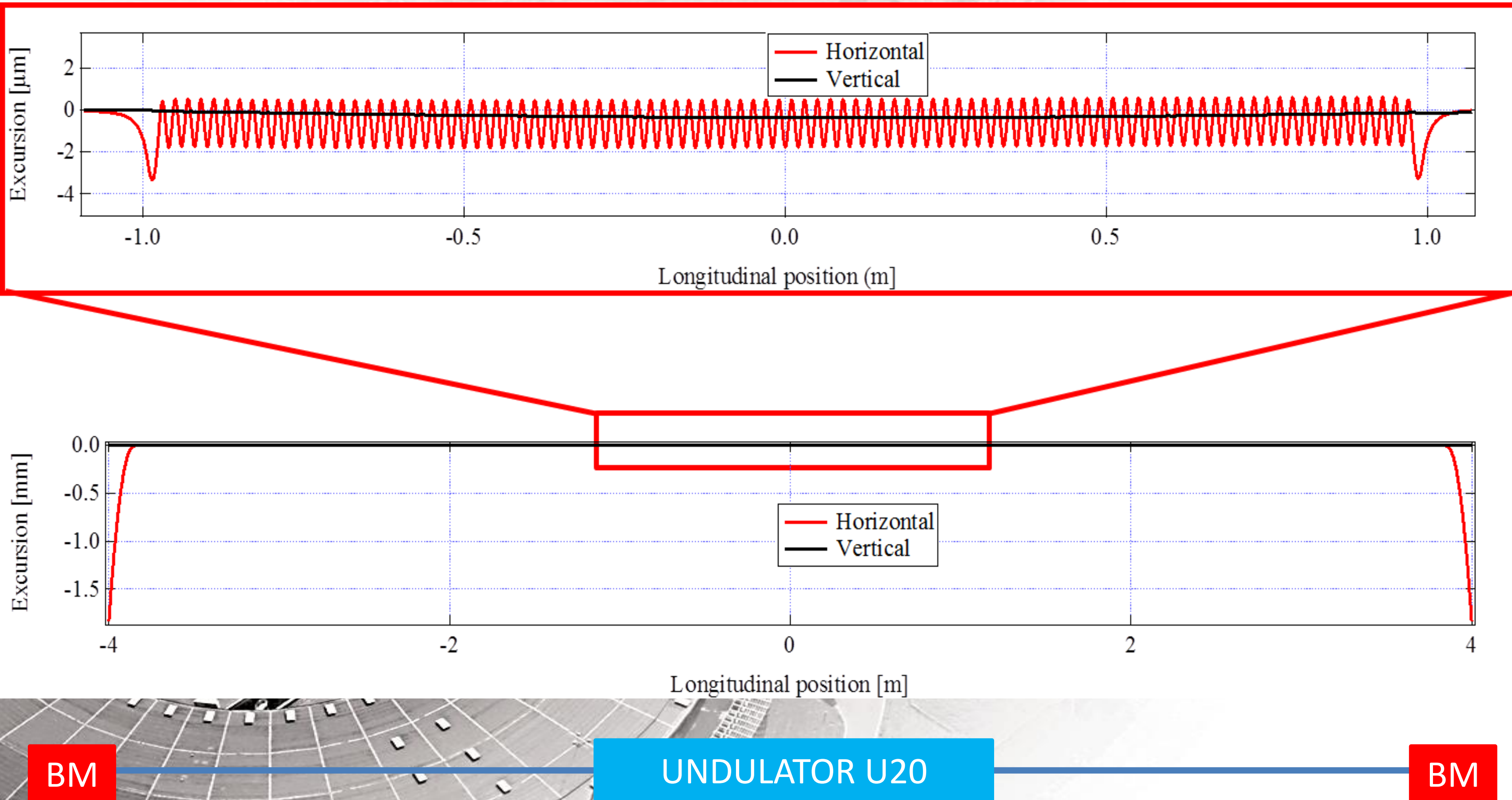


Short Model of U20*

Parameter	Value	Unit
Beam energy	2.75	GeV
Twiss parameters	Filament beam	-
Short straight section	7.722	m
Bending magnet field	1.7	T
Undulator type	In-vacuum U20 with NdFeB permanent magnets	-
Undulator period	20	mm
Period number	98+terminations	-
Magnetic gap	5.5	mm
Magnetic Field Harmonic content	B1: 0.9328 B3: 0.0617 B5: 0.0017	T T T

*RADIA Code

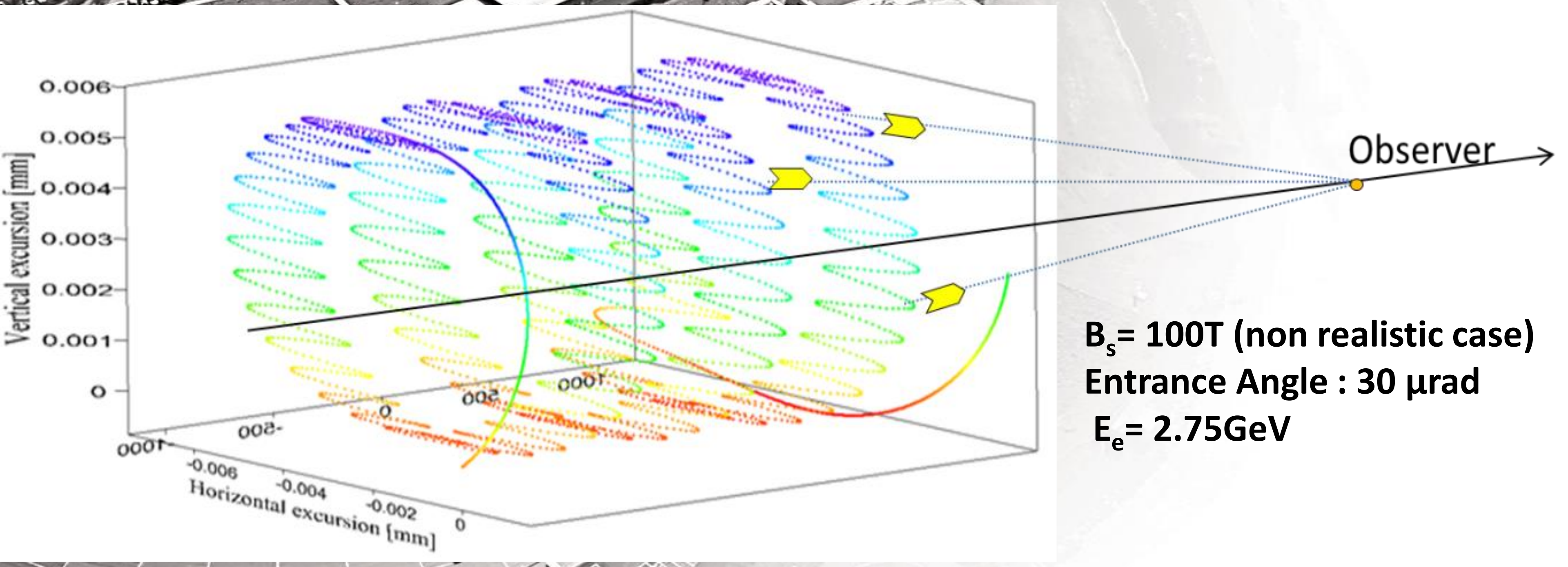
Adding a longitudinal field component (3/6): Motion



SRW code

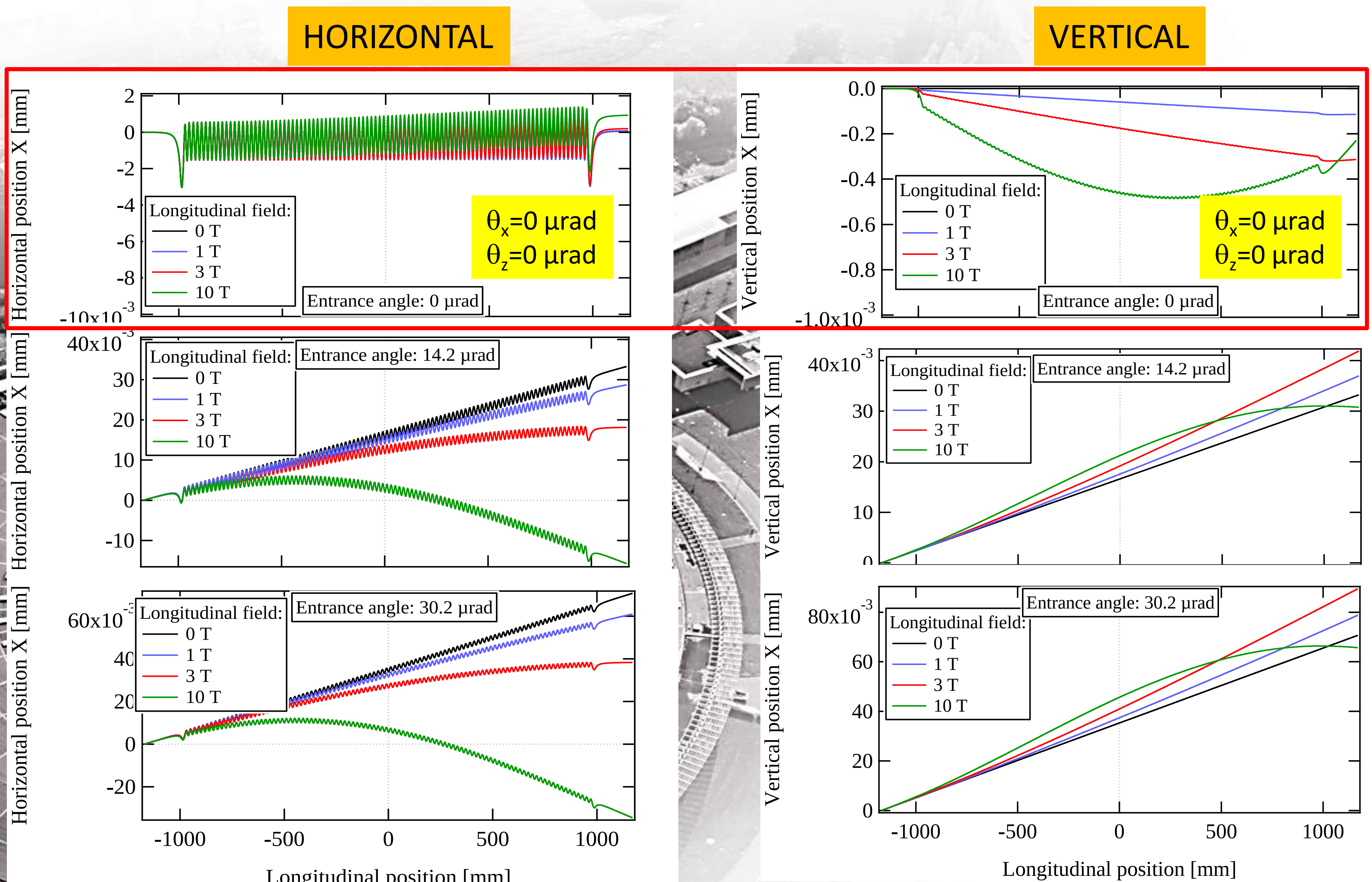
Adding a longitudinal field component (4/6): Motion

Although oscillating in the horizontal plane, electron beam rotate around the undulator axis and describes a helix at the precession pulsation $\omega = \frac{eB_s}{\gamma m_0}$ that is with a step length of $\Delta L = \frac{2\pi 10^9 E_e[GeV]}{cB_s}$. The amplitude of the helix depends on the longitudinal field amplitude and entrance angle.



B_s= 100T (non realistic case)
Entrance Angle : 30 µrad
E_e= 2.75GeV

Adding a longitudinal field component (5/6): Motion vs entrance angle



Adding a longitudinal field component (6/6): Flux density vs entrance angle

Electron beam injected on axis:

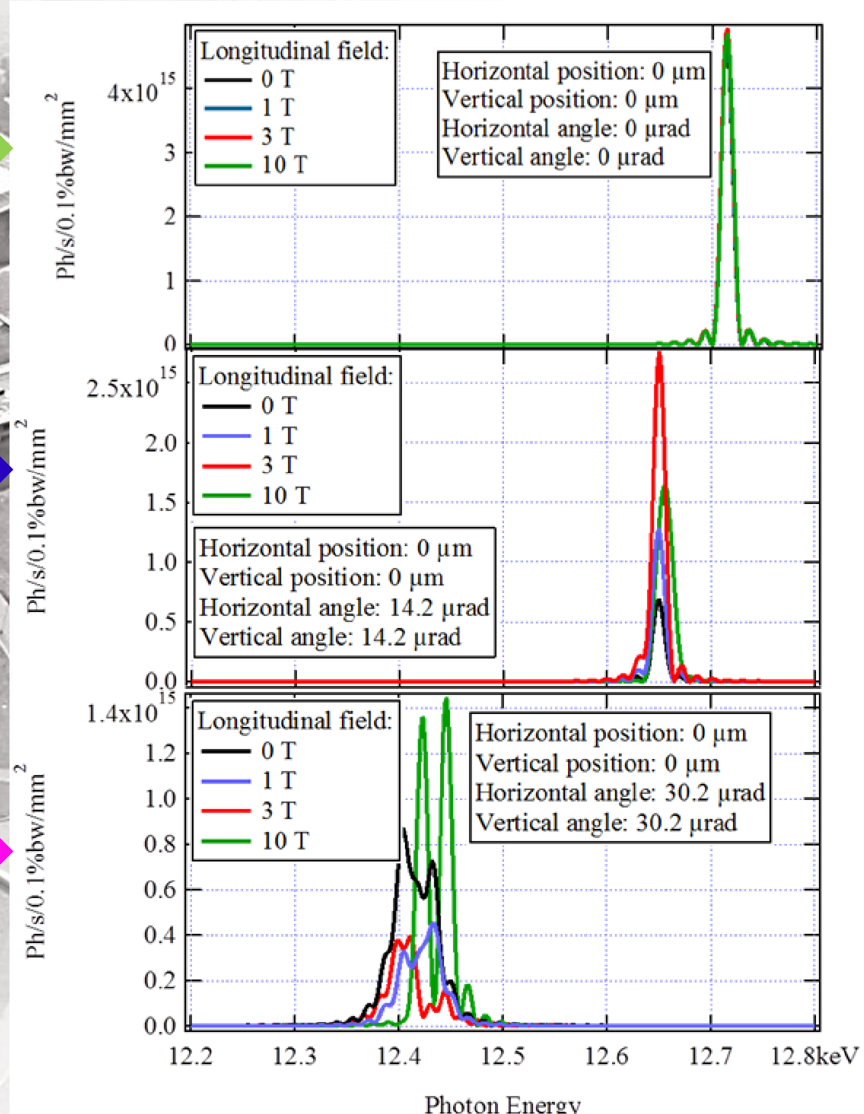
- θ_x=0 µrad
- θ_z=0 µrad

Electron beam injected with no-zero entrance angle:

- θ_x=14.2 µrad
- θ_z=14.2 µrad

Electron beam injected with no-zero entrance angle:

- θ_x=30.2 µrad
- θ_z=30.2 µrad

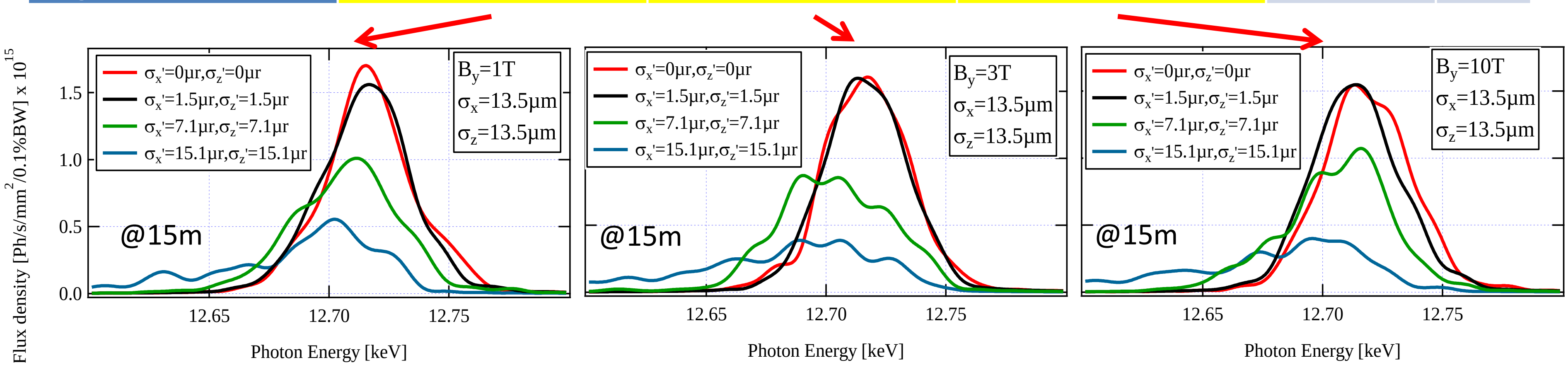


Flux density from a real beam

To take into account the emittance and the energy spread of the beam, one considers electrons distributed gaussianly independently in position, angular divergence and energy:

$$P(x,x',z,z',E) = P(x).P(x').P(z).P(z').P(E) \; with \; P(\xi) = \frac{1}{\sqrt{2\pi}\sigma_\xi} e^{\frac{1}{2}\left(\frac{\xi-\mu_\xi}{\sigma_\xi}\right)^2}$$

Parameter	Upgraded SOLEIL												Present SOLEIL	Unit
Energy E	2.75													GeV
Energy spread	0.101													%
Hor. Size σ_x	13.5												350	µm
Vert. size σ_z	13.5												9.6	µm
Hor. Divergence $\sigma_{x'}$	0	1.5	7.1	15.1	0	1.5	7.1	15.1	0	1.5	7.1	15.1	16.5	µr
Vert. divergence $\sigma_{z'}$	0	1.5	7.1	15.1	0	1.5	7.1	15.1	0	1.5	7.1	15.1	4	µr
Long. Field	1				3				10				0	T



NO SIGNIFICANT IMPACT OF THE AMPLITUDE OF LONGITUDINAL FIELD

Summary and conclusions

- Undulator radiation of composed of series of lines (harmonics)
- The amplitude and the bandwith of the harmonics strongly depends on the beam optics: emittance, energy spread, betatron and dispersion functions.
- Changes in the particle trajectory:
  - Magnetic defaults: → intensity reduction and bandwidth increase
  - Adding a longitudinal field: → No significant impact on the spectral performances