Emittance Control in a Normal Orthogonal Coordinate

- Flat or Round Electron Beams versus Magnetic Lattice of Storage Rings -

Hitoshi Tanaka and Kouichi Soutome
RIKEN SPring-8 Center

Outline

1. Introduction
2. Beam motion in a normal orthogonal coordinate
3. To utilize a full-transversal (Hori. x Vert.) phase space
4. Brilliance versus coupling ratio
5. Summary

1. Introduction -Where we are now-

ε_e: Natural electron beam emittance

ε_r: Photon emittance 9.9 pmrad@10 keV

When the condition is $\varepsilon_e << \varepsilon_r$ or $\varepsilon_e >> \varepsilon_r$, brilliance does not closely depend on the electron beam phase space distribution.

At ε_e ranging from several nmrad to 100 pmrad, which the current SR source can provide, <u>brilliance depends on the HV coupling ratio</u> and beta functions.

1. Introduction -Simple interpretation-

In the normal orthogonal system, the sum of horizontal and vertical emittance values is kept constant, e.g., A,

Let's try to understand the essence by a simple model where $X = (A-B) \times B = AB - B^2$ represents the transverse phase volume and 1/X corresponds to Brilliance.

100% coupling condition, A-B = B =1/2 A gives the maximum X and the minimum 1/X and

0% coupling condition, B=0 gives the minimum X and the maximum 1/X.

This is a simple answer to the question, why the existing SR source always operated with a small coupling ratio.

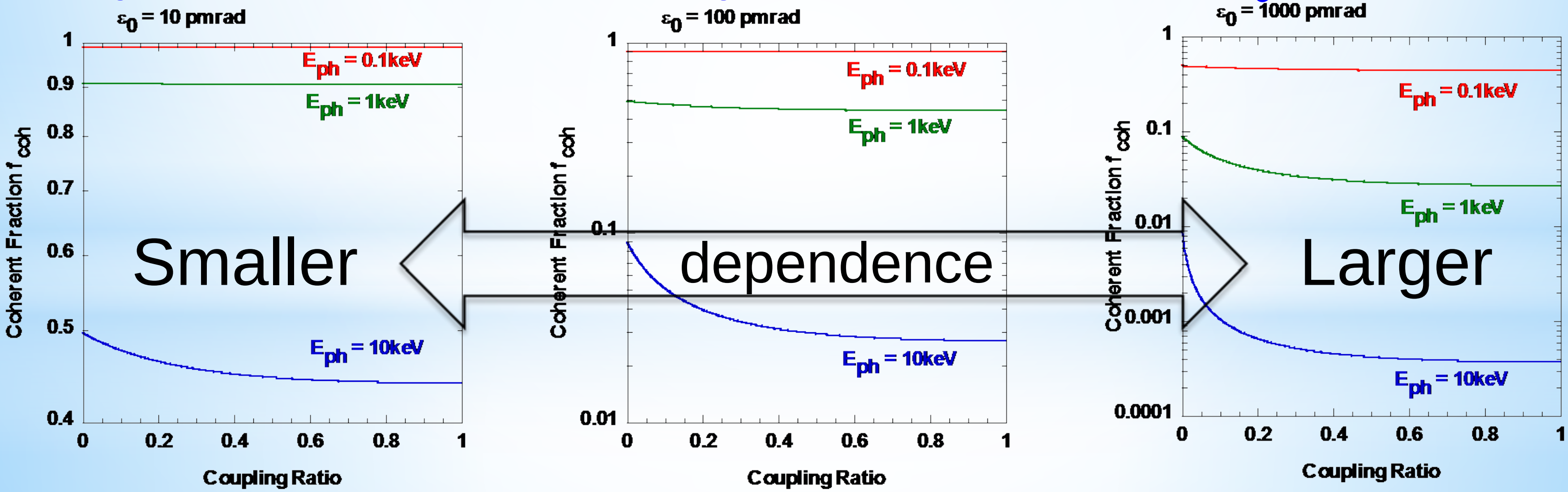
1. Introduction -Dependence-

Coherent Fraction

$$f_{coh} \equiv p_x p_y$$

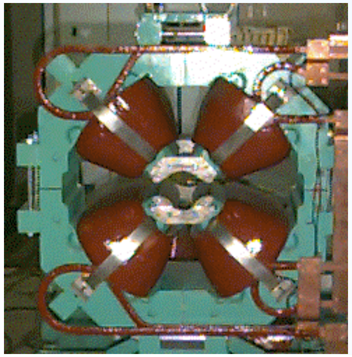
where $$p_x = \frac{\frac{\lambda}{4\pi}}{\sqrt{\sigma_r^2 + \sigma_x^2}\sqrt{\sigma_{r'}^2 + \sigma_{x'}^2}} \qquad p_y = \frac{\frac{\lambda}{4\pi}}{\sqrt{\sigma_r^2 + \sigma_y^2}\sqrt{\sigma_{r'}^2 + \sigma_{y'}^2}}$$

ε_0 = 10pmrad **ε_0 = 100pmrad** **ε_0 = 1000pmrad**

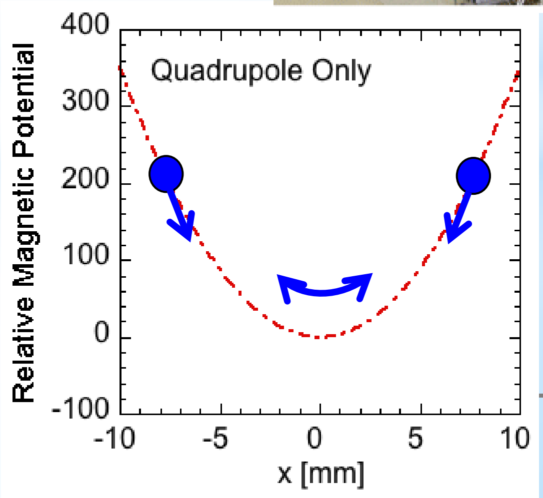


2. Beam motion in a normal orthogonal coordinate

We need potential wells which stabilize electron beam motion around the orbit to assure stable circulations in a storage ring.



Quadrupole magnets generate the adequate potential wells for two transversal oscillation modes, which are called betatron oscillations in the horizontal and vertical planes.



RF acceleration cavities generate the adequate potential well for longitudinal oscillation mode, which is called a synchrotron oscillation.

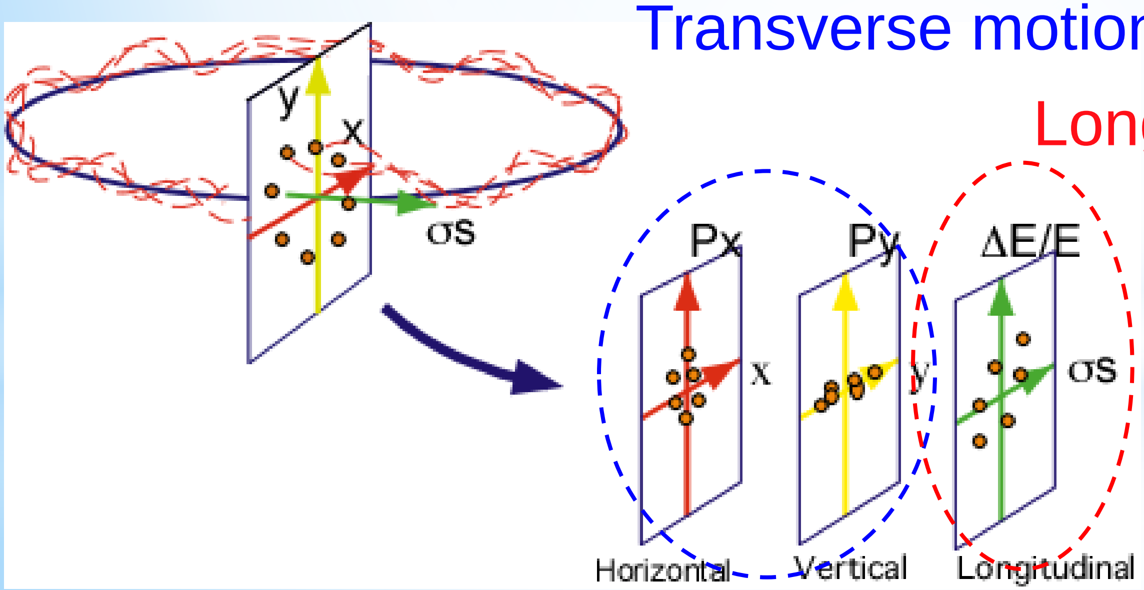
2. Beam motion in a normal orthogonal coordinate

6D-phase space volume of a single electron, ε_6 comprises of canonical variables (x, px(x'), y, py(y'), t, ps(ΔE/E)).

In an ideal case, 6D-phase space volume can be decomposed into three decoupled 2D phase spaces ε_h, h=x,y,s.

Transverse motion

Longitudinal motion



$$\varepsilon_6 = \varepsilon_x \varepsilon_y \varepsilon_s$$

2. Beam motion in a normal orthogonal coordinate

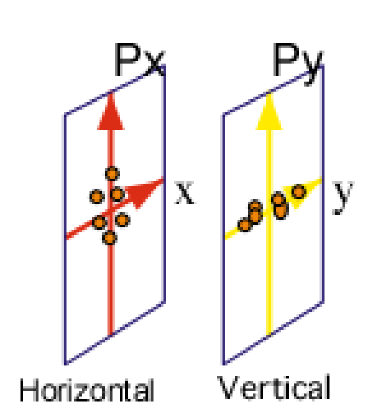
In the orthogonal system, we can only manipulate the transverse phase space keeping two eigenmodes; horizontal and vertical ones.

$\varepsilon_x \varepsilon_y$, full-transversal (Hori. x Vert.) phase space volume is not accessible in the orthogonal coordinate.

Stable manipulation is a differential resonance basis; $\varepsilon_x + \varepsilon_y$ = constant.

By controlling energy-transfer between the two modes, we can control the coupling ratio, $\varepsilon_y / \varepsilon_x$.

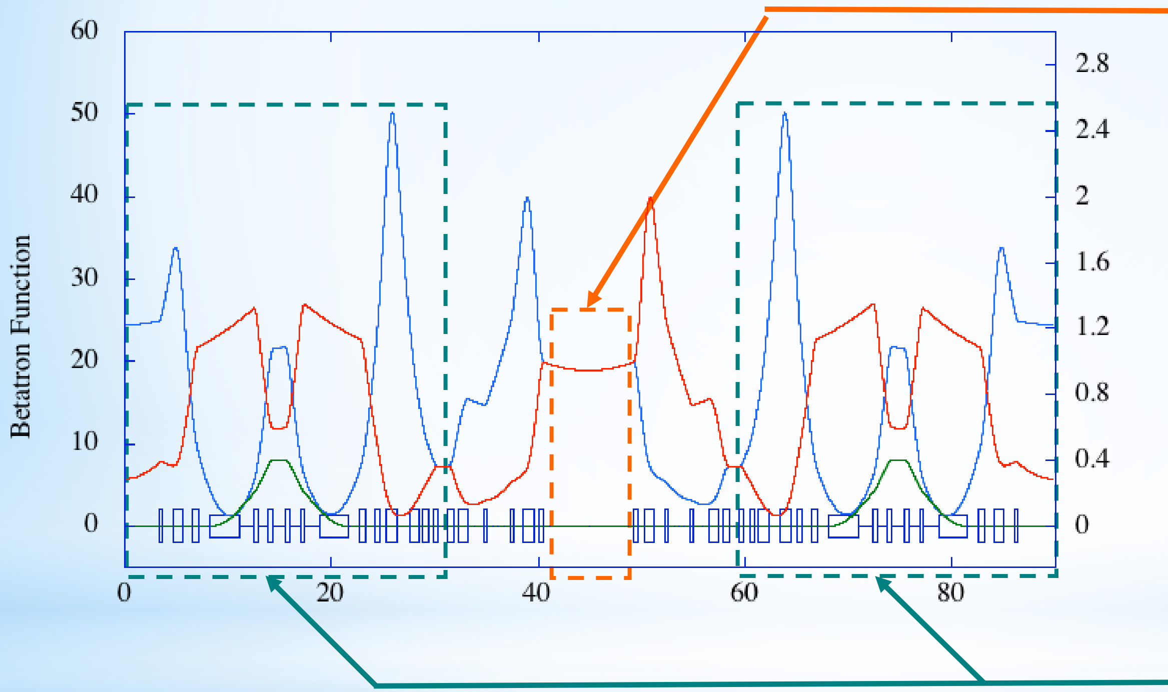
Transverse 2D decoupled phase spaces

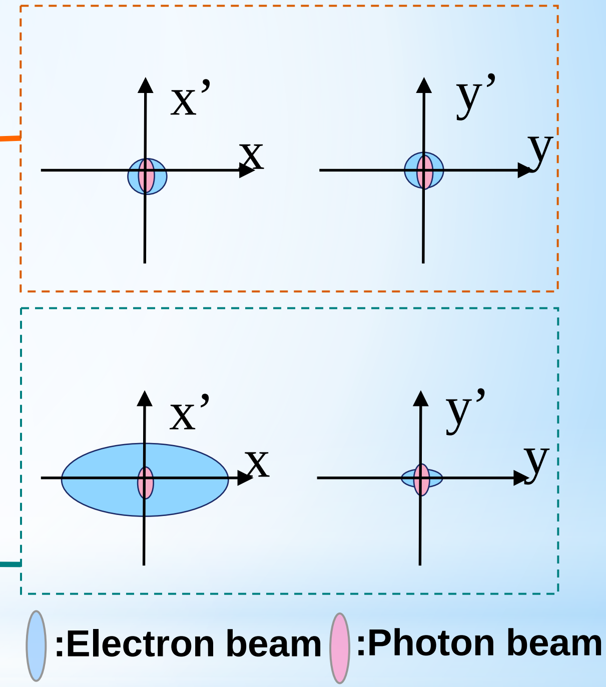


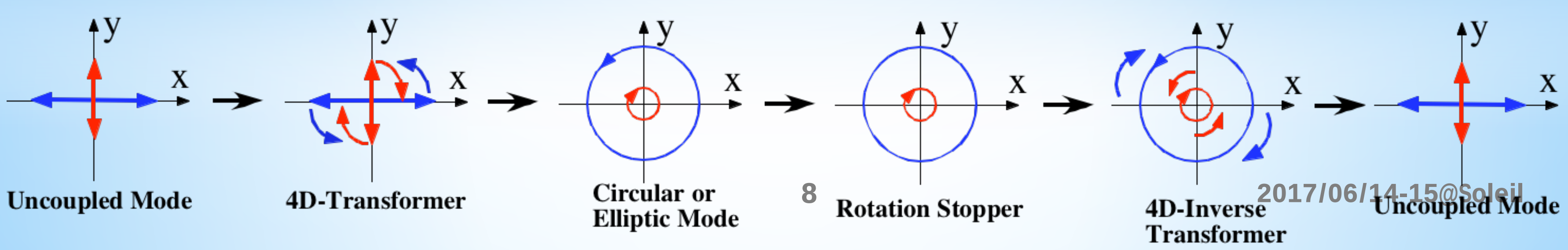
leil

3. To utilize a full-transversal phase space

Phase space adapter gives a "vortex mode" of which emittance is ~$\sqrt{\varepsilon_x \varepsilon_y}$.







4. Brilliance vs. Coupling Ratio
- Achievable HV coupling ratio -

HV coupling correction has been well established at 3rd. Gen. SR sources.

Thanks to

- development of alignment technology,
- advancement of analysis and correction methods based on the beam response,
- availability of high performance single pass BPM and photon beam diagnostic system

a smaller vertical emittance of several to a few tens pm.rad (a coupling ratio of 0.1%) is currently obtainable.

4. Brilliance vs. Coupling Ratio

Brilliance: $$B = \frac{Flux}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}} \equiv \frac{Flux}{4\pi^2 \Lambda_{xx'} \Lambda_{yy'}}$$

Gaussian distributions for electrons and photons:

$$\Lambda_{xx'} \equiv \Sigma_x \Sigma_{x'} = \sqrt{\sigma_r^2 + \sigma_x^2}\sqrt{\sigma_{r'}^2 + \sigma_{x'}^2} \qquad \Lambda_{yy'} \equiv \Sigma_y \Sigma_{y'} = \sqrt{\sigma_r^2 + \sigma_y^2}\sqrt{\sigma_{r'}^2 + \sigma_{y'}^2}$$

Electrons @ ID

$$\sigma_x = \sqrt{\beta_x \varepsilon_x} \quad \sigma_{x'} = \sqrt{\varepsilon_x / \beta_x} \quad \sigma_y = \sqrt{\beta_y \varepsilon_y} \quad \sigma_{y'} = \sqrt{\varepsilon_y / \beta_y}$$

$$\varepsilon_x = \frac{\varepsilon_0}{1+\kappa} \qquad \varepsilon_y = \frac{\kappa \varepsilon_0}{1+\kappa}$$

$\kappa = \varepsilon_y / \varepsilon_x$: Coupling Ratio $(0 < \kappa \le 1)$

Photons

$$\sigma_r = \frac{\sqrt{2L\lambda}}{4\pi} \qquad \sigma_{r'} = \sqrt{\frac{\lambda}{2L}} \qquad \varepsilon_r \equiv \sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$$

4. Brilliance vs. Coupling Ratio

$$B = \frac{Flux}{4\pi^2 \Lambda_{xx'} \Lambda_{yy'}}$$

with $\Lambda_{xx'} = \sqrt{\varepsilon_x^2 + \varepsilon_r^2 + \varepsilon_x \varepsilon_r \left(\frac{2\pi\beta_x}{L} + \frac{L}{2\pi\beta_x} \right)}$ $\Lambda_{yy'} = \sqrt{\varepsilon_y^2 + \varepsilon_r^2 + \varepsilon_y \varepsilon_r \left(\frac{2\pi\beta_y}{L} + \frac{L}{2\pi\beta_y} \right)}$

Brilliance is maximized at $\beta_x = \beta_y = \dfrac{L}{2\pi} \equiv \beta^{(opt)}$

$$B^{(\max)} = \frac{Flux}{4\pi^2 \Lambda^{(\min)}{}_{xx'} \Lambda^{(\min)}{}_{yy'}}$$

with $\Lambda^{(\min)}{}_{xx'} = \varepsilon_x + \varepsilon_r = \dfrac{1}{1+\kappa}\varepsilon_0 + \varepsilon_r$ $\Lambda^{(\min)}{}_{yy'} = \varepsilon_y + \varepsilon_r = \dfrac{\kappa}{1+\kappa}\varepsilon_0 + \varepsilon_r$

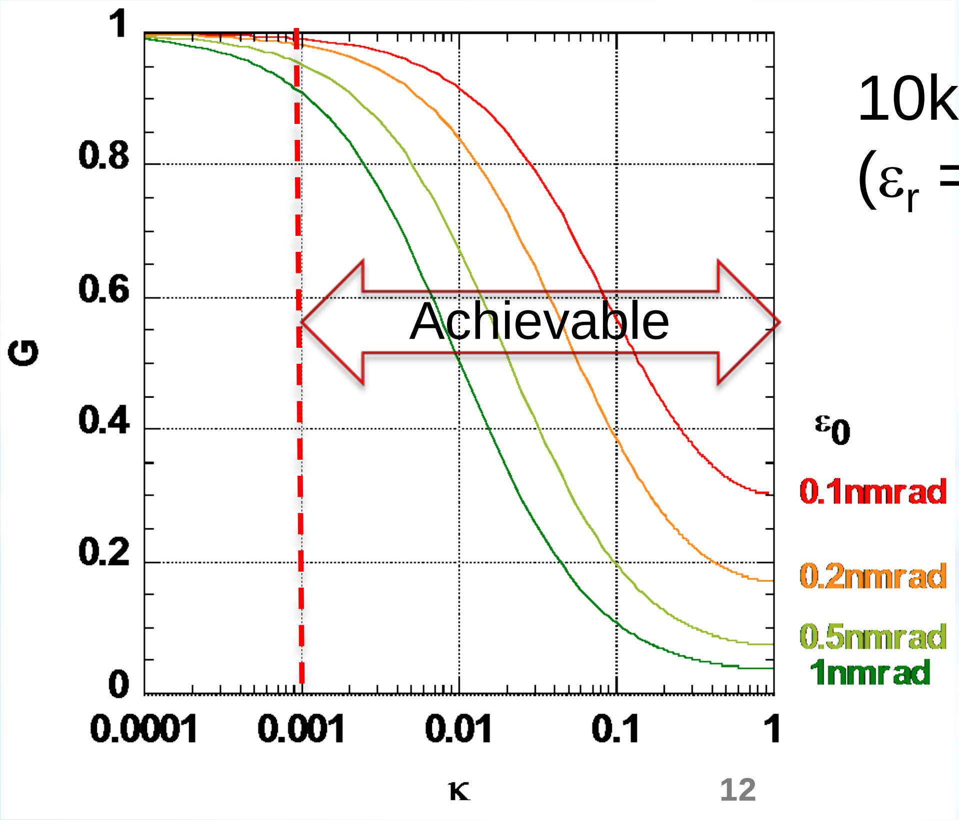
$$\frac{B}{B^{(\max)}} = \frac{\Lambda^{(\min)}{}_{xx'}}{\Lambda_{xx'}} \times \frac{\Lambda^{(\min)}{}_{yy'}}{\Lambda_{yy'}}$$

4. Brilliance vs. Coupling Ratio

κ-dependence of max. brilliance (for optimized beta)

$$B^{(\max)} = \frac{Flux}{4\pi^2 \Lambda^{(\min)}{}_{xx'} \Lambda^{(\min)}{}_{yy'}} = \frac{Flux}{4\pi^2 \varepsilon_r \left(\varepsilon_0 + \varepsilon_r\right)} G$$

$$G \equiv \frac{1}{1 + \frac{\kappa}{(1+\kappa)^2} \frac{\varepsilon_0^2}{\varepsilon_r \left(\varepsilon_0 + \varepsilon_r\right)}}$$



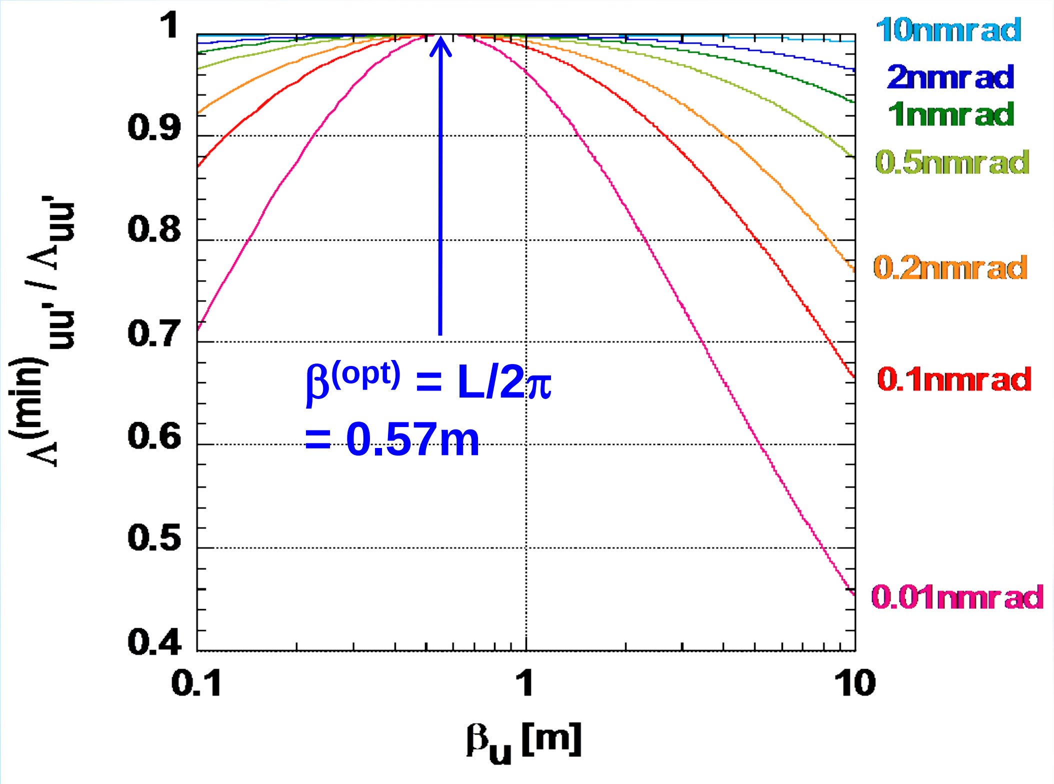
10keV photon
(ε_r = 9.9pmrad)

4. Brilliance vs. Coupling Ratio

Beta-dependence of brilliance

$$\frac{B}{B^{(\max)}} = \frac{\Lambda^{(\min)}{}_{xx'}}{\Lambda_{xx'}} \times \frac{\Lambda^{(\min)}{}_{yy'}}{\Lambda_{yy'}} \qquad \frac{\Lambda^{(\min)}{}_{uu'}}{\Lambda_{uu'}} \quad (u = x, y)$$

Phase space factor



L = 3.6m
10keV photon
(ε_r = 9.9pmrad)

For the emittance around 100pmrad, the phase factor or the brilliance depends on the beta values at ID.

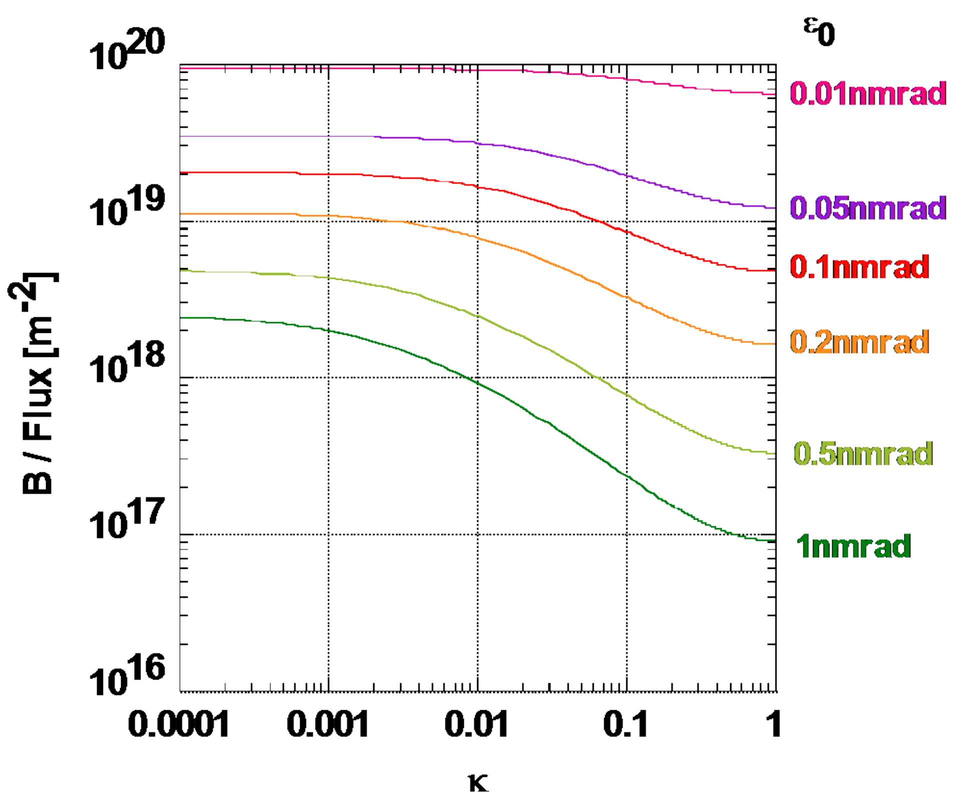
5. Summary

- A lower coupling condition is advantageous for achieving higher brilliance, as long as we adopt the normal orthogonal system for the emittance range from several to 0.1 nmrad currently available.
- As the emittance decreases, the sensitivity to the coupling ratio goes down, but the sensitivity to beta functions is enhanced.

*Spare Slide

4. Brilliance vs. Coupling Ratio

κ-dependence of brilliance $$B = B^{(\max)} \times \frac{\Lambda^{(\min)}{}_{xx'}}{\Lambda_{xx'}} \times \frac{\Lambda^{(\min)}{}_{yy'}}{\Lambda_{yy'}}$$



L = 3.6m

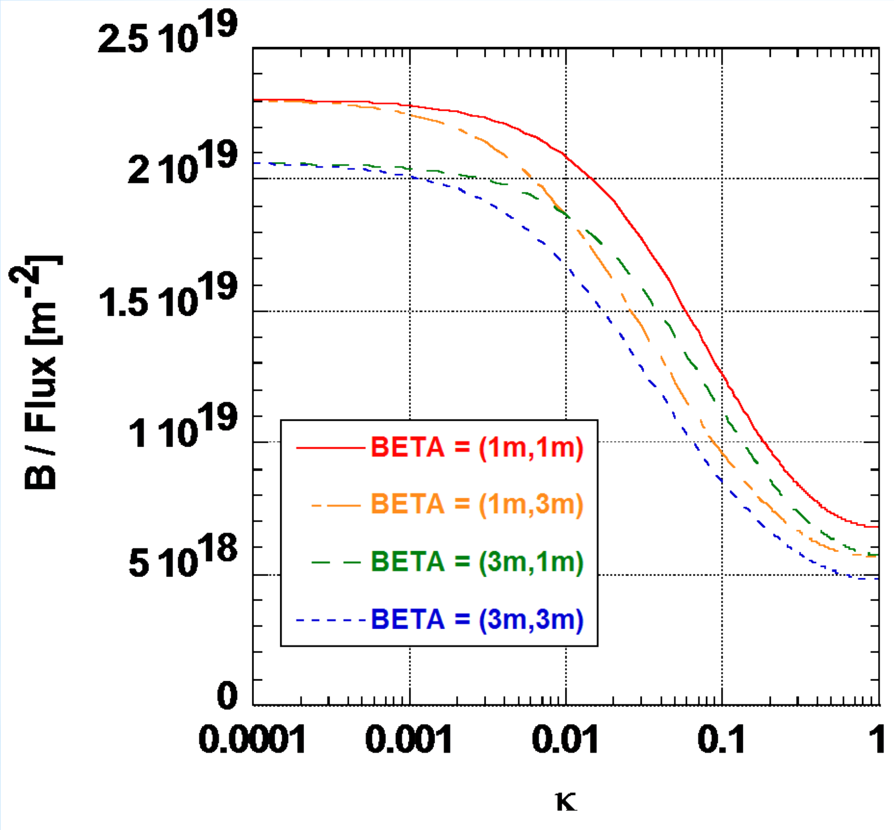
β_x = 3m, β_y = 3m

10keV photon

(ε_r = 9.9pmrad)

4. Brilliance vs. Coupling Ratio

κ-dependence of brilliance $$B = B^{(\max)} \times \frac{\Lambda^{(\min)}{}_{xx'}}{\Lambda_{xx'}} \times \frac{\Lambda^{(\min)}{}_{yy'}}{\Lambda_{yy'}}$$



L = 3.6m

ε_0 = 0.1nmrad

10keV photon
(ε_r = 9.9pmrad)